

thm_2Erich__list_2EREPLICATE__GENLIST
(TMYz72Y2UsGDpgK3yC6yHA14WV6b9DM7mj8)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (P \Rightarrow Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) (\text{c_2Ebool_2E_2F } V0t)))$

Definition 7 We define `c_2Ecombin_2E_2K` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 8 We define `c_2Ecombin_2E_2o` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } (\text{ty_2Enum_2Enum } A) \quad (2)$$

Let `c_2Elist_2EGENLIST__AUX` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2EGENLIST_AUX } A_27a \in \text{(((ty_2Elist_2Elist } A_27a)^{\text{ty_2Enum_2Enum } A_27a})^{\text{ty_2Enum_2Enum}})^{A_27a^{\text{ty_2Enum_2Enum}}}} \quad (3)$$

Definition 9 We define `c_2Earithmic_2ENUMERAL` to be $\lambda V0x \in \text{ty_2Enum_2Enum}. V0x$.

Let $c_2Elist_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EGENLIST\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{ty_2Enum_2Enum})^{(A_27a)^{ty_2Enum_2Enum}}) \quad (4)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (5)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Erich_list_2EREPLICATE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2EREPLICATE\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{A_27a})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (ap\ (c.2Ecombin_2EK \\ & A.27a\ A.27b)\ V0x)\ V1y) = V0x))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\ & A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow ((\forall V0f \in (A.27b^{A.27a}). \\ & (\forall V1v \in A.27c.((ap\ (ap\ (c.2Ecombin_2Eo\ A.27a\ A.27c\ A.27b) \\ & (ap\ (c.2Ecombin_2EK\ A.27c\ A.27b)\ V1v))\ V0f) = (ap\ (c.2Ecombin_2EK \\ & A.27c\ A.27a)\ V1v)))) \wedge (\forall V2f \in (A.27e^{A.27d}).(\forall V3v \in \\ & A.27d.((ap\ (ap\ (c.2Ecombin_2Eo\ A.27f\ A.27e\ A.27d)\ V2f)\ (ap\ (c.2Ecombin_2EK \\ & A.27d\ A.27f)\ V3v)) = (ap\ (c.2Ecombin_2EK\ A.27e\ A.27f)\ (ap\ V2f\ V3v)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a0 \in A.27a.(\forall V1a1 \in \\ & (ty_2Elist_2Elist\ A.27a).(\forall V2a0.27 \in A.27a.(\forall V3a1.27 \in \\ & (ty_2Elist_2Elist\ A.27a).(((ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V0a0) \\ & V1a1) = (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V2a0.27)\ V3a1.27)) \Leftrightarrow ((V0a0 = \\ & V2a0.27) \wedge (V1a1 = V3a1.27)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (A.27a^{ty_2Enum_2Enum}). \\ & (\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ (c.2Elist_2EGENLIST\ A.27a) \\ & V0f)\ (ap\ c.2Enum_2ESUC\ V1n)) = (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ (\\ & ap\ V0f\ c.2Enum_2E0))\ (ap\ (ap\ (c.2Elist_2EGENLIST\ A.27a)\ (ap\ (ap \\ & (c.2Ecombin_2Eo\ ty_2Enum_2Enum\ A.27a\ ty_2Enum_2Enum)\ V0f)\ c.2Enum_2ESUC)) \\ & V1n)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (A_27a^{ty_2Enum_2Enum}). \\
& (\forall V1n \in ty_2Enum_2Enum. (((ap\ (ap\ (c_2Elist_2EGENLIST\ A_27a) \\
& V0f)\ c_2Enum_2E0) = (c_2Elist_2ENIL\ A_27a)) \wedge ((ap\ (ap\ (c_2Elist_2EGENLIST \\
& A_27a)\ V0f)\ (ap\ c_2Earithmetic_2ENUMERAL\ V1n)) = (ap\ (ap\ (ap\ (c_2Elist_2EGENLIST_AUX \\
& A_27a)\ V0f)\ (ap\ c_2Earithmetic_2ENUMERAL\ V1n))\ (c_2Elist_2ENIL \\
& A_27a))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((ap\ (ap\ (c_2Erich_list_2EREPLICATE \\
& A_27a)\ c_2Enum_2E0)\ V0x) = (c_2Elist_2ENIL\ A_27a)) \wedge (\forall V1n \in \\
& ty_2Enum_2Enum. (\forall V2x \in A_27a. ((ap\ (ap\ (c_2Erich_list_2EREPLICATE \\
& A_27a)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2x) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a) \\
& V2x)\ (ap\ (ap\ (c_2Erich_list_2EREPLICATE\ A_27a)\ V1n)\ V2x))))))
\end{aligned} \tag{22}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \forall V1x \in A_27a. ((ap\ (ap\ (c_2Erich_list_2EREPLICATE\ A_27a) \\
& V0n)\ V1x) = (ap\ (ap\ (c_2Elist_2EGENLIST\ A_27a)\ (ap\ (c_2Ecombin_2EK \\
& A_27a\ ty_2Enum_2Enum)\ V1x))\ V0n))))
\end{aligned}$$