

thm\_2Erich\_\_list\_2ESEG\_\_APPEND2  
(TMHDC4SijuhnchQWCm6fidRygdEo5XujKm4)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (ap c\_2Enum\_2EREP\_num (ap c\_2Enum\_2ESUC\_REP m)))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_Emin\_2E\_40$

**Definition 11** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 13** We define  $c\_Earithmic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (6)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (7)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ELENGTH A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist A\_27a)}) \quad (8)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (9)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (10)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (11)$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Erich\_list\_2ESEG : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Erich\_list\_2ESEG A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\ & (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0n)) (ap \\ & c\_2Enum\_2ESUC V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) \\ & V1m)))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0n)) c\_2Enum\_2E0)))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2D c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge ((ap (ap c\_2Earithmetic\_2E\_2D V0m) c\_2Enum\_2E0) = V0m))) \quad (15)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Enum\_2ESUC V0n)) (ap c\_2Enum\_2ESUC V1m)) = (ap (ap c\_2Earithmetic\_2E\_2D V0n) V1m))) \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (19)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (22)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p V0t))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\
& A.27a).((ap (ap (c.2Elist\_2EAPPEND A.27a) (c.2Elist\_2ENIL A.27a)) \\
& V0l) = V0l) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist A.27a).(\forall V2l2 \in \\
& (ty\_2Elist\_2Elist A.27a).(\forall V3h \in A.27a.((ap (ap (c.2Elist\_2EAPPEND \\
& A.27a) (ap (ap (c.2Elist\_2ECONS A.27a) V3h) V1l1)) V2l2) = (ap (ap \\
& (c.2Elist\_2ECONS A.27a) V3h) (ap (ap (c.2Elist\_2EAPPEND A.27a) \\
& V1l1) V2l2))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (((ap (c.2Elist\_2ELENGTH A.27a) \\
& (c.2Elist\_2ENIL A.27a)) = c.2Enum\_2E0) \wedge (\forall V0h \in A.27a.( \\
& \forall V1t \in (ty\_2Elist\_2Elist A.27a).((ap (c.2Elist\_2ELENGTH \\
& A.27a) (ap (ap (c.2Elist\_2ECONS A.27a) V0h) V1t)) = (ap c.2Enum\_2ESUC \\
& (ap (c.2Elist\_2ELENGTH A.27a) V1t))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist A.27a)}). \\
& (((p (ap V0P (c.2Elist\_2ENIL A.27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\
& A.27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A.27a.(p (ap V0P (ap (ap ( \\
& c.2Elist\_2ECONS A.27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& A.27a).(p (ap V0P V3l))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c.2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c.2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0k \in ty\_2Enum\_2Enum. \\
& (\forall V1l \in (ty\_2Elist\_2Elist\ A.27a).((ap\ (ap\ (ap\ (c.2Erich\_list\_2ESEG \\
& A.27a)\ c.2Enum\_2E0)\ V0k)\ V1l) = (c.2Elist\_2ENIL\ A.27a)))) \wedge ((\forall V2m \in \\
& ty\_2Enum\_2Enum.(\forall V3x \in A.27a.(\forall V4l \in (ty\_2Elist\_2Elist \\
& A.27a).((ap\ (ap\ (ap\ (c.2Erich\_list\_2ESEG\ A.27a)\ (ap\ c.2Enum\_2ESUC \\
& V2m))\ c.2Enum\_2E0)\ (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a)\ V3x)\ V4l)) = \\
& (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a)\ V3x)\ (ap\ (ap\ (ap\ (c.2Erich\_list\_2ESEG \\
& A.27a)\ V2m)\ c.2Enum\_2E0)\ V4l)))))) \wedge (\forall V5m \in ty\_2Enum\_2Enum. \\
& (\forall V6k \in ty\_2Enum\_2Enum.(\forall V7x \in A.27a.(\forall V8l \in \\
& (ty\_2Elist\_2Elist\ A.27a).((ap\ (ap\ (ap\ (c.2Erich\_list\_2ESEG \\
& A.27a)\ (ap\ c.2Enum\_2ESUC\ V5m))\ (ap\ c.2Enum\_2ESUC\ V6k))\ (ap\ (ap\ ( \\
& c.2Elist\_2ECONS\ A.27a)\ V7x)\ V8l)) = (ap\ (ap\ (ap\ (c.2Erich\_list\_2ESEG \\
& A.27a)\ (ap\ c.2Enum\_2ESUC\ V5m))\ V6k)\ V8l)))))))))
\end{aligned} \tag{29}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\
& A.27a).(\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3l2 \in (ty\_2Elist\_2Elist\ A.27a).(((p\ (ap\ (ap\ c.2Earithmetic\_2E\_3C\_3D \\
& (ap\ (c.2Elist\_2ELENGTH\ A.27a)\ V0l1))\ V1m)) \wedge (p\ (ap\ (ap\ c.2Earithmetic\_2E\_3C\_3D \\
& V2n)\ (ap\ (c.2Elist\_2ELENGTH\ A.27a)\ V3l2)))))) \Rightarrow ((ap\ (ap\ (ap\ (c.2Erich\_list\_2ESEG \\
& A.27a)\ V2n)\ V1m)\ (ap\ (ap\ (c.2Elist\_2EAPPEND\ A.27a)\ V0l1)\ V3l2)) = \\
& (ap\ (ap\ (ap\ (c.2Erich\_list\_2ESEG\ A.27a)\ V2n)\ (ap\ (ap\ c.2Earithmetic\_2E\_2D \\
& V1m)\ (ap\ (c.2Elist\_2ELENGTH\ A.27a)\ V0l1)))\ V3l2))))))
\end{aligned}$$