

thm_2Erich__list_2ESEG__LASTN__BUTLASTN
(TMTBrJS-
CLDx9DhWieoLwPqcLLQgEX5PtDbk)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{6}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \text{omega} \quad (7)$$

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A.\lambda 27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V2t1 \in A.$

Definition 17 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 18 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 19 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 20 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 21 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 22 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (11)$$

Let $c_2Erich_list_2ESEG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Erich_list_2ESEG\ A.27a \in ((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)}_{ty_2Enum_2Enum})_{ty_2Enum_2Enum} \quad (12)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2EREVERSE\ A.27a \in ((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)}) \quad (13)$$

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2EDROP\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})_{ty_2Enum_2Enum}) \quad (14)$$

Definition 23 We define $c_2Erich_list_2EBUTLASTN$ to be $\lambda A.27a : \iota.\lambda V0n \in ty_2Enum_2Enum.\lambda V1xs$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ETAKE\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})_{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})_{ty_2Enum_2Enum} \quad (16)$$

Definition 24 We define $c_2Erich_list_2ELASTN$ to be $\lambda A.27a : \iota.\lambda V0n \in ty_2Enum_2Enum.\lambda V1xs \in (ty$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ELENGTH\ A.27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist\ A.27a)} \quad (17)$$

Definition 25 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge (((ap\ (\\ & ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2B \\ & (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))))))))) \end{aligned} \quad (18)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (19)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \quad (20)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (21)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \quad (23)$$

Assume the following.

$$(\forall V0a \in ty_2Enum_2Enum.(\forall V1b \in ty_2Enum_2Enum.(\forall V2c \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V0a) (ap (ap c_2Earithmetic_2E_2B V1b) V2c)) = (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2D V0a) V1b)) V2c)))))) \quad (24)$$

Assume the following.

$$p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.2D } V0n) V1m)) V0n))) \quad (25)$$

Assume the following.

$$(\forall V0m \in ty.\text{2Enum.2Enum.} (\forall V1n \in ty.\text{2Enum.2Enum.} (V0m = V1n) \Leftrightarrow ((p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } V0m) V1n)) \wedge (p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } V1n) V0m)))))) \quad (26)$$

Assume the following.

$$(\forall V0m \in ty.\text{2Enum.2Enum.} (\forall V1n \in ty.\text{2Enum.2Enum.} (\forall V2p \in ty.\text{2Enum.2Enum.} ((p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.2B } V0m) V1n)) (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.2B } V0m) V2p))) \Leftrightarrow (p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } V1n) V2p)))))) \quad (27)$$

Assume the following.

$$(\forall V0m \in ty.\text{2Enum.2Enum.} (\forall V1n \in ty.\text{2Enum.2Enum.} (\neg (p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } V0m) V1n))) \Leftrightarrow (p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } (\text{ap } c.\text{2Enum.2ESUC } V1n)) V0m)))) \quad (28)$$

Assume the following.

$$(\forall V0m \in ty.\text{2Enum.2Enum.} (\forall V1n \in ty.\text{2Enum.2Enum.} (\neg (V0m = V1n)) \Leftrightarrow ((p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } (\text{ap } c.\text{2Enum.2ESUC } V0m)) V1n)) \vee (p (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.3C.3D } (\text{ap } c.\text{2Enum.2ESUC } V1n)) V0m)))))) \quad (29)$$

Assume the following.

$$(\forall V0n \in ty.\text{2Enum.2Enum.} ((\text{ap } c.\text{2Enum.2ESUC } V0n) = (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.2B } (\text{ap } c.\text{2Earithmetic.2ENUMERAL } (\text{ap } c.\text{2Earithmetic.2EBIT1 } c.\text{2Earithmetic.2EZERO})) V0n))) \quad (30)$$

Assume the following.

$$(\forall V0P \in (2^{ty.\text{2Enum.2Enum}}). (\forall V1a \in ty.\text{2Enum.2Enum.} (\forall V2b \in ty.\text{2Enum.2Enum.} ((p (\text{ap } V0P (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.2D } V1a) V2b))) \Leftrightarrow (\forall V3d \in ty.\text{2Enum.2Enum.} (((V2b = (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.2B } V1a) V3d)) \Rightarrow (p (\text{ap } V0P c.\text{2Enum.2E0})) \wedge ((V1a = (\text{ap } (\text{ap } c.\text{2Earithmetic.2E.2B } V2b) V3d)) \Rightarrow (p (\text{ap } V0P V3d)))))))))) \quad (31)$$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge ((p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ((p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p \\ & V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \forall V1l \in (ty_2Elist_2Elist A_27a). ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) (ap (c_2Elist_2ELENGTH A_27a) V1l))) \Rightarrow ((ap (c_2Elist_2ELENGTH \\
& A_27a) (ap (ap (c_2Erich_list_2ELASTN A_27a) V0n) V1l)) = V0n))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \forall V1m \in ty_2Enum_2Enum. (\forall V2l \in (ty_2Elist_2Elist \\
& A_27a). ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B \\
& V0n) V1m)) (ap (c_2Elist_2ELENGTH A_27a) V2l))) \Rightarrow ((ap (ap (c_2Erich_list_2ELASTN \\
& A_27a) V0n) (ap (ap (c_2Erich_list_2EBUTLASTN A_27a) V1m) V2l)) = \\
& (ap (ap (c_2Erich_list_2EBUTLASTN A_27a) V1m) (ap (ap (c_2Erich_list_2ELASTN \\
& A_27a) (ap (ap c_2Earithmetic_2E_2B V0n) V1m)) V2l))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \forall V1m \in ty_2Enum_2Enum. (\forall V2l \in (ty_2Elist_2Elist \\
& A_27a). ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B \\
& V0n) V1m)) (ap (c_2Elist_2ELENGTH A_27a) V2l))) \Rightarrow ((ap (ap (ap (c_2Erich_list_2ESEG \\
& A_27a) V0n) V1m) V2l) = (ap (ap (c_2Elist_2ETAKE A_27a) V0n) (ap (\\
& ap (c_2Elist_2EDROP A_27a) V1m) V2l))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\ & \forall V1l \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & V0n)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l))) \Rightarrow ((ap\ (ap\ (c_2Elist_2ETAKE \\ & A_27a)\ V0n)\ V1l) = (ap\ (ap\ (c_2Erich_list_2EBUTLASTN\ A_27a)\ (ap \\ & (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l)) \\ & V0n))\ V1l)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\ & \forall V1l \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & V0n)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l))) \Rightarrow ((ap\ (ap\ (c_2Elist_2EDROP \\ & A_27a)\ V0n)\ V1l) = (ap\ (ap\ (c_2Erich_list_2ELASTN\ A_27a)\ (ap\ (ap \\ & c_2Earithmetic_2E_2D\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l))\ V0n)) \\ & V1l)))))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
& \hspace{20em} (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
& \hspace{20em} (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \\
& \hspace{20em} (61)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \quad \forall V1m \in ty_2Enum_2Enum. (\forall V2l \in (ty_2Elist_2Elist \\
& \quad A.27a). ((p (ap (ap c.2Earithmetic.2E_3C_3D (ap (ap c.2Earithmetic.2E_2B \\
V0n) V1m)) (ap (c.2Elist.2ELENGTH A.27a) V2l))) \Rightarrow ((ap (ap (ap (c.2Erich_list.2ESEG \\
& \quad A.27a) V0n) V1m) V2l) = (ap (ap (c.2Erich_list.2ELASTN A.27a) V0n) \\
& \quad (ap (ap (c.2Erich_list.2EBUTLASTN A.27a) (ap (ap c.2Earithmetic.2E_2D \\
& \quad (ap (c.2Elist.2ELENGTH A.27a) V2l)) (ap (ap c.2Earithmetic.2E_2B \\
& \quad V0n) V1m))) V2l))))))
\end{aligned}$$