

thm_2Erich__list_2ESUM__FLAT (TM- FGTxEx8Xo1DvBPKX5groEuq16fUxC6GX3u)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{4}$$

Let $c_2Elist_2EFLAT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFLAT\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A_27a))}) \tag{5}$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a}})) \quad (6)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (7)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (8)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (10)$$

Let $c_2Elist_2ESUM : \iota$ be given. Assume the following.

$$c_2Elist_2ESUM \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ ty_2Enum_2Enum)}) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (15)$$

Assume the following.

$$\begin{aligned}
& (((ap\ c_2Elist_2ESUM\ (c_2Elist_2ENIL\ ty_2Enum_2Enum)) = c_2Enum_2E0) \wedge \\
& \quad (\forall V0h \in ty_2Enum_2Enum. (\forall V1t \in (ty_2Elist_2Elist \\
& \quad ty_2Enum_2Enum). ((ap\ c_2Elist_2ESUM\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad ty_2Enum_2Enum)\ V0h)\ V1t)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0h) \\
& \quad (ap\ c_2Elist_2ESUM\ V1t))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2EFLAT\ A_27a)\ (\\
& \quad c_2Elist_2ENIL\ (ty_2Elist_2Elist\ A_27a))) = (c_2Elist_2ENIL \\
& \quad A_27a)) \wedge (\forall V0h \in (ty_2Elist_2Elist\ A_27a). (\forall V1t \in \\
& \quad (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A_27a)). ((ap\ (c_2Elist_2EFLAT \\
& \quad A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Elist_2Elist\ A_27a)\ V0h) \\
& \quad V1t)) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0h)\ (ap\ (c_2Elist_2EFLAT \\
& \quad A_27a)\ V1t))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& \quad V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& \quad (A_27b^{A_27a}). (\forall V2h \in A_27a. (\forall V3t \in (ty_2Elist_2Elist \\
& \quad A_27a). ((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& \quad (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& \quad (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& \quad A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\
& \quad c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& \quad A_27a). (p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0l1 \in (ty_2Elist_2Elist\ ty_2Enum_2Enum). (\forall V1l2 \in \\
& \quad (ty_2Elist_2Elist\ ty_2Enum_2Enum). ((ap\ c_2Elist_2ESUM\ (ap\ (\\
& \quad ap\ (c_2Elist_2EAPPEND\ ty_2Enum_2Enum)\ V0l1)\ V1l2)) = (ap\ (ap\ c_2Earithmetic_2E_2B \\
& \quad (ap\ c_2Elist_2ESUM\ V0l1))\ (ap\ c_2Elist_2ESUM\ V1l2))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned}
& (\forall V0l \in (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ ty_2Enum_2Enum))). \\
& ((ap\ c_2Elist_2ESUM\ (ap\ (c_2Elist_2EFLAT\ ty_2Enum_2Enum)\ V0l)) = \\
& \quad (ap\ c_2Elist_2ESUM\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Elist_2Elist \\
& \quad ty_2Enum_2Enum)\ ty_2Enum_2Enum)\ c_2Elist_2ESUM)\ V0l)))
\end{aligned}$$