

thm_2Erich__list_2ESUM__FOLDR
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October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{5}$$

Let $c_2Elist_2ESUM : \iota$ be given. Assume the following.

$$c_2Elist_2ESUM \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ ty_2Enum_2Enum)}) \tag{6}$$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDR\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b^{A_27b})^{A_27a})}) \tag{7}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)(ty_2Elist_2Elist\ A_27a))^{A_27a})^{A_27a} \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (9)$$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\begin{aligned} & (((ap\ c_2Elist_2ESUM\ (c_2Elist_2ENIL\ ty_2Enum_2Enum)) = c_2Enum_2E0) \wedge \\ & (\forall V0h \in ty_2Enum_2Enum.(\forall V1t \in (ty_2Elist_2Elist\ ty_2Enum_2Enum). \\ & (ap\ c_2Elist_2ESUM\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Enum_2Enum)\ V0h)\ V1t)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0h)\ (ap\ c_2Elist_2ESUM\ V1t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0f \in ((A_27b^{A_27b})^{A_27a}).(\forall V1e \in A_27b.((ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL\ A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27b})^{A_27a}).(\forall V3e \in A_27b. \\ & (\forall V4x \in A_27a.(\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\ & ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V2f)\ V3e)\ V5l))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a). \\ & ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (14)$$

Theorem 1

$$(\forall V0l \in (ty_2Elist_2Elist\ ty_2Enum_2Enum).((ap\ c_2Elist_2ESUM\ V0l) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDER\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ c_2Earithmic_2E_2B)\ c_2Enum_2E0)\ V0l)))$$