

thm\_2Erich\_\_list\_2ETAKE (TMGrBHkELcKEjk-  
bgmtTQGFhT1KXXc5HLkD6)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 10** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 11** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 12** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (8)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (9)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (10)$$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 15** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ c\_2Enum\_2ESUC\ V0m))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) = V0m)) \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p \ V0t))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0a0 \in A\_27a.(\forall V1a1 \in \\
& (ty\_2Elist\_2Elist \ A\_27a).(\forall V2a0\_27 \in A\_27a.(\forall V3a1\_27 \in \\
& (ty\_2Elist\_2Elist \ A\_27a).(((ap \ (ap \ (c\_2Elist\_2ECONS \ A\_27a) \ V0a0) \\
& V1a1) = (ap \ (ap \ (c\_2Elist\_2ECONS \ A\_27a) \ V2a0\_27) \ V3a1\_27)) \Leftrightarrow ((V0a0 = \\
& V2a0\_27) \wedge (V1a1 = V3a1\_27))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.( \\
& \forall V1x \in A\_27a.(\forall V2xs \in (ty\_2Elist\_2Elist \ A\_27a).( \\
& (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ c\_2Enum\_2E0) \ V0n)) \Rightarrow ((ap \ (ap \ (c\_2Elist\_2ETAKE \\
& A\_27a) \ V0n) \ (ap \ (ap \ (c\_2Elist\_2ECONS \ A\_27a) \ V1x) \ V2xs)) = (ap \ (ap \\
& (c\_2Elist\_2ECONS \ A\_27a) \ V1x) \ (ap \ (ap \ (c\_2Elist\_2ETAKE \ A\_27a) \ ( \\
& ap \ (ap \ c\_2Earithmetic\_2E\_2D \ V0n) \ (ap \ c\_2Earithmetic\_2ENUMERAL \\
& (ap \ c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO)))))) \ V2xs))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
& A\_27a).((ap \ (ap \ (c\_2Elist\_2ETAKE \ A\_27a) \ c\_2Enum\_2E0) \ V0l) = (c\_2Elist\_2ENIL \\
& A\_27a)))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ c\_2Enum\_2E0) \\
& (ap \ c\_2Enum\_2ESUC \ V0n))))
\end{aligned} \tag{20}$$

**Theorem 1**

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\ & A_{27a}). ((ap (ap (c\_2Elist\_2ETAKE A_{27a}) c\_2Enum\_2E0) V0l) = (c\_2Elist\_2ENIL \\ & A_{27a}))) \wedge (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2x \in A_{27a}. ( \\ & \forall V3l \in (ty\_2Elist\_2Elist A_{27a}). ((ap (ap (c\_2Elist\_2ETAKE \\ & A_{27a}) (ap c\_2Enum\_2ESUC V1n)) (ap (ap (c\_2Elist\_2ECONS A_{27a}) \\ & V2x) V3l)) = (ap (ap (c\_2Elist\_2ECONS A_{27a}) V2x) (ap (ap (c\_2Elist\_2ETAKE \\ & A_{27a}) V1n) V3l)))))))))) \end{aligned}$$