thm_2Erich_list_2ETAKE__SEG (TMRKABfLkaEx6YpyFFrfAEbFcwxHvgPSff1)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty \; ty\_2Enum\_2Enum \qquad (1)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (4)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

1

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{5}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$\tag{6}$$

Let $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})$$
$$\tag{7}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$\tag{8}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$
$$\tag{9}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{10}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Erich\_\_list\_2ESEG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Erich\_\_list\_2ESEG\ A\_27a \in ((\\((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$\tag{11}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ c\_2Enum\_2ESUC\ V0n))\ (ap\\c\_2Enum\_2ESUC\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0n)\\V1m)))))$$
$$\tag{12}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))\ \mathsf{c\_2Enum\_2E0})))) \tag{13}$$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p$$
$$V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow$$
$$True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((\tag{18}$$
$$(p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t))))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow$$
$$True)) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)$$
$$(c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0})\wedge(\forall V0h \in A\_27a.($$
$$\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH$$
$$A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).$$
$$((((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a)))\wedge(\forall V1t \in (ty\_2Elist\_2Elist$$
$$A\_27a).((p\ (ap\ V0P\ V1t))\Rightarrow(\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ($$
$$c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))))\Rightarrow(\forall V3l \in (ty\_2Elist\_2Elist$$
$$A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{21}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum})).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0}))\wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n))\Rightarrow(p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))$$

(22)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((ap\ (ap\ (c\_2Elist\_2ETAKE\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0l) = (c\_2Elist\_2ENIL$$
$$A\_27a)))\wedge(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2x \in A\_27a.($$
$$\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2ETAKE$$
$$A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)$$
$$V2x)\ V3l)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ (ap\ (ap\ (c\_2Elist\_2ETAKE$$
$$A\_27a)\ V1n)\ V3l)))))))$$

(23)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0k \in ty\_2Enum\_2Enum.$$
$$(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Erich\_list\_2ESEG$$
$$A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0k)\ V1l) = (c\_2Elist\_2ENIL\ A\_27a))))\wedge((\forall V2m \in$$
$$ty\_2Enum\_2Enum.(\forall V3x \in A\_27a.(\forall V4l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((ap\ (ap\ (ap\ (c\_2Erich\_list\_2ESEG\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V2m))\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3x)\ V4l)) =$$
$$(ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3x)\ (ap\ (ap\ (ap\ (c\_2Erich\_list\_2ESEG$$
$$A\_27a)\ V2m)\ \mathsf{c\_2Enum\_2E0})\ V4l))))))\wedge(\forall V5m \in ty\_2Enum\_2Enum.$$
$$(\forall V6k \in ty\_2Enum\_2Enum.(\forall V7x \in A\_27a.(\forall V8l \in$$
$$(ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Erich\_list\_2ESEG$$
$$A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V5m))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V6k))\ (ap\ (ap\ ($$
$$c\_2Elist\_2ECONS\ A\_27a)\ V7x)\ V8l)) = (ap\ (ap\ (ap\ (c\_2Erich\_list\_2ESEG$$
$$A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V5m))\ V6k)\ V8l)))))))))$$

(24)

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0n \in ty\_2Enum\_2Enum.($$
$$\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l)))\Rightarrow((ap\ (ap\ (c\_2Elist\_2ETAKE$$
$$A\_27a)\ V0n)\ V1l) = (ap\ (ap\ (ap\ (c\_2Erich\_list\_2ESEG\ A\_27a)\ V0n)$$
$$\mathsf{c\_2Enum\_2E0})\ V1l)))))$$

4