

thm_2Erich__list_2ETAKE__SNOC (TMH- FWRX3ND6JB9MqEBGJFVMzeeHibdRjjX8)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (ap c_2Enum_2EREP_num (ap c_2Enum_2ESUC_REP m)))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (5)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist A_27a)}) \quad (6)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ESNOC A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (7)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ETAKE A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0n)) (ap \\ & c_2Enum_2ESUC V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) \\ & V1m)))))) \end{aligned} \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (((ap (c_2Elist_2ELENGTH A_27a) (c_2Elist_2ENIL A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2ELENGTH A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = (ap c_2Enum_2ESUC (ap (c_2Elist_2ELENGTH A_27a) V1t)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A.27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0x \in A.27a.((ap\ (ap\ (c_2Elist_2ESNOC \\ & A.27a)\ V0x)\ (c_2Elist_2ENIL\ A.27a)) = (ap\ (ap\ (c_2Elist_2ECONS \\ & A.27a)\ V0x)\ (c_2Elist_2ENIL\ A.27a)))) \wedge (\forall V1x \in A.27a.(\forall V2x.27 \in \\ & A.27a.(\forall V3l \in (ty_2Elist_2Elist\ A.27a).((ap\ (ap\ (c_2Elist_2ESNOC \\ & A.27a)\ V1x)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V2x.27)\ V3l)) = (ap\ (\\ & ap\ (c_2Elist_2ECONS\ A.27a)\ V2x.27)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A.27a) \\ & V1x)\ V3l)))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ c_2Enum_2E0)))) \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\ & A.27a).((ap\ (ap\ (c_2Elist_2ETAKE\ A.27a)\ c_2Enum_2E0)\ V0l) = (c_2Elist_2ENIL \\ & A.27a))) \wedge (\forall V1n \in ty_2Enum_2Enum.(\forall V2x \in A.27a.(\\ & \forall V3l \in (ty_2Elist_2Elist\ A.27a).((ap\ (ap\ (c_2Elist_2ETAKE \\ & A.27a)\ (ap\ c_2Enum_2ESUC\ V1n))\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a) \\ & V2x)\ V3l)) = (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V2x)\ (ap\ (ap\ (c_2Elist_2ETAKE \\ & A.27a)\ V1n)\ V3l)))))) \end{aligned} \quad (27)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ & \forall V1l \in (ty_2Elist_2Elist\ A.27a).((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\ & V0n)\ (ap\ (c_2Elist_2ELENGTH\ A.27a)\ V1l))) \Rightarrow (\forall V2x \in A.27a. \\ & ((ap\ (ap\ (c_2Elist_2ETAKE\ A.27a)\ V0n)\ (ap\ (ap\ (c_2Elist_2ESNOC \\ & A.27a)\ V2x)\ V1l)) = (ap\ (ap\ (c_2Elist_2ETAKE\ A.27a)\ V0n)\ V1l)))))) \end{aligned}$$