

thm_2Erich__list_2ETAKE__TAKE (TMListqzY- LAycARv8Wvw9dTkGC3RRdAqqgC)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{2}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2. V0x))\ (\lambda V1x \in 2. V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (V0P))))$

Definition 4 We define $c_2Ebool_2E_F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. V2t))))$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ETAKE\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{omega}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (7)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge p\ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ P))))$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Eprim_rec_2E_3C\ m\ n)$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (V0t1\ V1t2))))$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Earithmetic_2E_3C_3D\ m\ n)$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t\ x) \Leftrightarrow (p\ V0t\ x)))) \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p\ V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (14)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.((((p \ V0x) \Leftrightarrow (p \ V1x_{.27})) \wedge ((p \ V1x_{.27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{.27})))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{.27}) \Rightarrow (p \ V3y_{.27})))))) \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\\ & \forall V1l \in (ty_2Elist_2Elist \ A_{.27a}).(\forall V2n \in ty_2Enum_2Enum. \\ & ((p \ (ap \ (ap \ c_2Earithmetic_2E_3C_3D \ V2n) \ V0m)) \Rightarrow ((ap \ (ap \ (c_2Elist_2ETAKE \\ & \ A_{.27a}) \ V2n) \ (ap \ (ap \ (c_2Elist_2ETAKE \ A_{.27a}) \ V0m) \ V1l)) = (ap \ (ap \ (\\ & \ c_2Elist_2ETAKE \ A_{.27a}) \ V2n) \ V1l)))))) \end{aligned} \quad (16)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\\ & \forall V1l \in (ty_2Elist_2Elist \ A_{.27a}).((p \ (ap \ (ap \ c_2Earithmetic_2E_3C_3D \\ & \ V0m) \ (ap \ (c_2Elist_2ELENGTH \ A_{.27a}) \ V1l))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. \\ & ((p \ (ap \ (ap \ c_2Earithmetic_2E_3C_3D \ V2n) \ V0m)) \Rightarrow ((ap \ (ap \ (c_2Elist_2ETAKE \\ & \ A_{.27a}) \ V2n) \ (ap \ (ap \ (c_2Elist_2ETAKE \ A_{.27a}) \ V0m) \ V1l)) = (ap \ (ap \ (\\ & \ c_2Elist_2ETAKE \ A_{.27a}) \ V2n) \ V1l)))))) \end{aligned}$$