

thm\_2Erich\_\_list\_2EZIP\_\_SNOC  
(TMKPAez4eTjVAHLmkSj83z4uT1QiND4Fvd9)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{1}$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Eenum\_2Eenum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{5}$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$   
Let  $c\_2Elist\_2EZIP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EZIP A\_27a A\_27b \in ((ty\_2Elist\_2Elist (ty\_2Epair\_2Eprod A\_27a A\_27b))^{(ty\_2Epair\_2Eprod (ty\_2Elist\_2Elist A\_27a A\_27b))}) \quad (6)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (7)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (8)$$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ESNOC A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (9)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (13)$$

**Definition 10** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(c\_2Enum\_2E0 = (ap c\_2Enum\_2ESUC V0n)))) \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ c\_2Enum\_2ESUC\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l)))))) \quad (24)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a0 \in A\_27a. (\forall V1a1 \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27a). (\forall V2a0\_27 \in A\_27a. (\forall V3a1\_27 \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27a). (((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0a0) \\
& \quad V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = \\
& \quad V2a0\_27) \wedge (V1a1 = V3a1\_27))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (((ap\ (c\_2Elist\_2EZIP \\
& \quad A\_27c\ A\_27d)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist\ A\_27c) \\
& \quad (ty\_2Elist\_2Elist\ A\_27d))\ (c\_2Elist\_2ENIL\ A\_27c))\ (c\_2Elist\_2ENIL \\
& \quad A\_27d))) = (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27d))) \wedge \\
& \quad (\forall V0x1 \in A\_27a. (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a). \\
& \quad (\forall V2x2 \in A\_27b. (\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27b). \\
& \quad ((ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist \\
& \quad A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a) \\
& \quad V0x1)\ V1l1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V2x2)\ V3l2))) = (ap \\
& \quad (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ ( \\
& \quad c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x1)\ V2x2))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a \\
& \quad A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist \\
& \quad A\_27b))\ V1l1)\ V3l2))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a. ((ap\ (ap\ (c\_2Elist\_2ESNOC \\
& \quad A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)) = (ap\ (ap\ (c\_2Elist\_2ECONS \\
& \quad A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)))) \wedge (\forall V1x \in A\_27a. (\forall V2x\_27 \in \\
& \quad A\_27a. (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (c\_2Elist\_2ESNOC \\
& \quad A\_27a)\ V1x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x\_27)\ V3l)) = (ap\ ( \\
& \quad ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x\_27)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a) \\
& \quad V1x)\ V3l))))))
\end{aligned} \tag{27}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg((ap\ c\_2Enum\_2ESUC\ V0n) = c\_2Enum\_2E0))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ c\_2Enum\_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \tag{29}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist \\ & A\_27b). (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH \\ & A\_27b)\ V1l2)) \Rightarrow (\forall V2x1 \in A\_27a. (\forall V3x2 \in A\_27b. ((ap \\ & (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist \\ & A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a) \\ & V2x1)\ V0l1))\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27b)\ V3x2)\ V1l2)))) = (ap \\ & (ap\ (c\_2Elist\_2ESNOC\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ ( \\ & c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2x1)\ V3x2))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a \\ & A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist \\ & A\_27b))\ V0l1)\ V1l2)))))))))) \end{aligned}$$