

thm_2Erich_list_2Elongest_prefix_PAIR
(TMQrAbbaPVu2soCb2GutzP64qHhCBHbJ5ok)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}})^{A_27b})^{(ty_2Elist_2Elist\ A_27a)}) \quad (2)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (3)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2E2 to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x)))$

Definition 4 We define $c_2Emarker_2EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (6)$$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ c_2Enum_2ESUC_REP\ m))$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A.\lambda y.y \in A))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda P \in 2^A.(\lambda V0P \in 2^A.(ap\ V0P\ (ap\ (c_2Emin_2E_40\ P))))$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ c_2Eprim_rec_2E_3C\ m\ n)$

Definition 13 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ c_2Earithmetic_2E_3E\ m\ n)$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 15 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ c_2Earithmetic_2E_3E_3D\ m\ n)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A.\lambda P \in 2^A.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.(ap\ P\ (V1t1\ V2t2))))$

Definition 18 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ P)\ m)\ n)\ o)$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (11)$$

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (12)$$

Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (13)$$

Definition 19 We define $c_Enumeral_EiZ$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Definition 20 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Earithmetic_E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (14)$$

Definition 21 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_Enum_Enum.(ap (ap c_Earithmetic_EBIT1) V0n)$.

Definition 22 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap (ap c_Earithmetic_EBIT0) V0n)$.

Definition 23 We define $c_Earithmetic_EZERO$ to be c_Enum_E0 .

Let $ty_Epair_Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_Epair_Eprod A0 A1) \quad (15)$$

Definition 24 We define $c_Earithmetic_E_3C_3D$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum.V0m$.

Definition 25 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$.

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EABS_prod A_27a A_27b \in ((ty_Epair_Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (16)$$

Definition 26 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_Epair_EABS_prod) V0x V1y)$.

Let $c_Epred_set_EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epred_set_EGSPEC A_27a A_27b \in ((2^{A_27a})^{((ty_Epair_Eprod A_27a 2)^{A_27b})}) \quad (17)$$

Definition 27 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in$

Definition 28 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap (c_2Epred_set_2E$

Definition 29 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2E$

Definition 30 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 31 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_21 (2$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist A_27a)}) \quad (18)$$

Definition 32 We define $c_2Epred_set_2Eis_measure_maximal$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Enum_2Enum$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EisPREFIX A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (19)$$

Definition 33 We define $c_2Erich_list_2Ecommon_prefixes$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Elist_2Elist A_27a)})$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (20)$$

Definition 34 We define $c_2Erich_list_2Elongest_prefix$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Elist_2Elist A_27a)})$.

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0n)) (ap \\ & c_2Enum_2ESUC V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) \\ & V1m)))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ & V0m) V1n)))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\ & ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\ & V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\ & V1n)) V0m)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(((ap c_2Enum_2ESUC V0n) = (ap (ap \\ & c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO))) V0n))) \end{aligned} \quad (29)$$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF \\ & V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Emin_2E_40 \\ & A_27a)\ (\lambda V1y \in A_27a.(ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V1y)\ V0x))) = \\ & V0x)) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & (2^{A_27a}).(((\exists V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (\forall V3x \in \\ & A_27a.((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ (ap\ V1Q\ V3x)))))) \Rightarrow (p\ (ap\ V1Q\ (ap\ (c_2Emin_2E_40 \\ & A_27a)\ V0P)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a.(p\ (\\ & ap\ V1Q\ V4x)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ & p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (52)$$

Assume the following.

$$(\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2R \in 2. (((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R)))))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow \text{False}) \Leftrightarrow ((p V0t) \Leftrightarrow \text{False}))) \quad (54)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (55)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (56)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}. (\forall V3x_{.27} \in A_{.27a}. (\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((\text{ap } (\text{ap } (\text{ap } (\text{c.2Ebool_2ECOND } A_{.27a}) \\ & V0P) V2x) V4y) = (\text{ap } (\text{ap } (\text{ap } (\text{c.2Ebool_2ECOND } A_{.27a}) V1Q) V3x_{.27} \\ & V5y_{.27})))))))))) \quad (58) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (59)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}). (\forall V1v \in A_27a. ((\forall V2x \in A_27a. ((V2x = V1v) \Rightarrow (p (ap\ V0f\ V2x)))) \Leftrightarrow (p (ap\ V0f\ V1v)))))) \quad (60)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap (ap (ap (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap (ap (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. (((\exists V2x \in A_27a. (p (ap\ V0P\ V2x))) \Rightarrow (p\ V1Q)) \Leftrightarrow (\forall V3x \in A_27a. ((p (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \wedge (((\exists V4x \in A_27a. (p (ap\ V0P\ V4x))) \wedge (p\ V1Q)) \Leftrightarrow (\exists V5x \in A_27a. ((p (ap\ V0P\ V5x)) \wedge (p\ V1Q)))))) \wedge (((p\ V1Q) \wedge (\exists V6x \in A_27a. (p (ap\ V0P\ V6x)))) \Leftrightarrow (\exists V7x \in A_27a. ((p\ V1Q) \wedge (p (ap\ V0P\ V7x))))))))) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (((ap (c_2Elist_2ELENGTH\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap (c_2Elist_2ELENGTH\ A_27a)\ (ap (ap (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC\ (ap (c_2Elist_2ELENGTH\ A_27a)\ V1t)))))) \quad (63)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a0 \in A_27a. (\forall V1a1 \in (ty_2Elist_2Elist\ A_27a). (\forall V2a0_27 \in A_27a. (\forall V3a1_27 \in (ty_2Elist_2Elist\ A_27a). (((ap (ap (c_2Elist_2ECONS\ A_27a)\ V0a0)\ V1a1) = (ap (ap (c_2Elist_2ECONS\ A_27a)\ V2a0_27)\ V3a1_27)) \Leftrightarrow ((V0a0 = V2a0_27) \wedge (V1a1 = V3a1_27)))))) \quad (64)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist\ A_27a). (\forall V1a0 \in A_27a. (\neg ((c_2Elist_2ENIL\ A_27a) = (ap (ap (c_2Elist_2ECONS\ A_27a)\ V1a0)\ V0a1)))))) \quad (65)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).((p (ap (ap (c_2Elist_2EisPREFIX A_27a) (c_2Elist_2ENIL \\
& A_27a)) V0l)) \Leftrightarrow True)) \wedge (\forall V1h \in A_27a.(\forall V2t \in (ty_2Elist_2Elist \\
& A_27a).(\forall V3l \in (ty_2Elist_2Elist A_27a).((p (ap (ap (c_2Elist_2EisPREFIX \\
& A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V1h) V2t)) V3l)) \Leftrightarrow (p (ap (\\
& ap (ap (c_2Elist_2Elist_CASE A_27a 2) V3l) c_2Ebool_2EF) (\lambda V4h_27 \in \\
& A_27a.(\lambda V5t_27 \in (ty_2Elist_2Elist A_27a).(ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap (c_2Emin_2E_3D A_27a) V1h) V4h_27)) (ap (ap (c_2Elist_2EisPREFIX \\
& A_27a) V2t) V5t_27))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\
& A_27a).(\forall V1y \in A_27a.(\forall V2ys \in (ty_2Elist_2Elist \\
& A_27a).((p (ap (ap (c_2Elist_2EisPREFIX A_27a) V0x) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V1y) V2ys))) \Leftrightarrow ((V0x = (c_2Elist_2ENIL A_27a)) \vee (\exists V3xs \in \\
& (ty_2Elist_2Elist A_27a).((V0x = (ap (ap (c_2Elist_2ECONS A_27a) \\
& V1y) V3xs)) \wedge (p (ap (ap (c_2Elist_2EisPREFIX A_27a) V3xs) V2ys))))))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& A_27b. (((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap \\
& (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN \\
& A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0f \in ((ty_2Epair_2Eprod A_27a 2)^{A_27b}). (\forall V1v \in \\
& A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V1v) (ap (c_2Epred_set_2EGSPEC \\
& A_27a A_27b) V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap (ap (c_2Epair_2E_2C \\
& A_27a 2) V1v) c_2Ebool_2ET) = (ap V0f V2x))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap \\
& (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (\forall V2s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN A_27a) \\
& V0x) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V2s))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).((V0s = \\ (c.2Epred_set.2EEMPTY\ A.27a)) \vee (\exists V1x \in A.27a.(\exists V2t \in \\ (2^{A-27a}).((V0s = (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V1x) \\ V2t)) \wedge (\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V2t)))))))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1s \in \\ (2^{A-27a}).(\neg((ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V0x)\ V1s) = \\ (c.2Epred_set.2EEMPTY\ A.27a)))))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ A.27a.(((ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V0x)\ (c.2Epred_set.2EEMPTY \\ A.27a)) = (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V1y)\ (c.2Epred_set.2EEMPTY \\ A.27a)))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0y \in A.27b.(\forall V1s \in (2^{A-27a}).(\forall V2f \in (A.27b^{A-27a}). \\ ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V0y)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE \\ A.27a\ A.27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A.27a.((V0y = (ap\ V2f\ V3x)) \wedge \\ (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V3x)\ V1s)))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1s \in \\ (2^{A-27a}).((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EINSERT \\ A.27a)\ V0x)\ V1s)))) \Leftrightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V1s)))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0s \in (2^{A-27a}).((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a) \\ V0s)) \Rightarrow (\forall V1f \in (A.27b^{A-27a}).(p\ (ap\ (c.2Epred_set.2EFINITE \\ A.27b)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27b)\ V1f)\ V0s)))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).((\\ \exists V1s \in (2^{A-27a}).((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A-27a})\ V1s) \\ V0P)) \wedge (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V1s)))) \Rightarrow (p\ (ap\ (c.2Epred_set.2EFINITE \\ A.27a)\ (ap\ (c.2Epred_set.2EBIGINTER\ A.27a)\ V0P)))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0m \in (ty_2Enum_2Enum^{A_27a}). \\
& \quad (\forall V1s \in (2^{A_27a}). ((p (ap (c_2Epred_set_2EFINITE A_27a) \\
& \quad V1s)) \wedge (\neg (V1s = (c_2Epred_set_2EEMPTY A_27a)))) \Rightarrow (\exists V2x \in \\
& \quad A_27a. (p (ap (ap (ap (ap (c_2Epred_set_2Eis_measure_maximal A_27a) \\
& \quad V0m) V1s) V2x))))))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0m \in (ty_2Enum_2Enum^{A_27a}). \\
& \quad (\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p (ap (ap (ap (c_2Epred_set_2Eis_measure_maximal \\
& \quad A_27a) V0m) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1x) (c_2Epred_set_2EEMPTY \\
& \quad A_27a))) V2y)) \Leftrightarrow (V2y = V1x))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\
& \quad (2^{A_27a}). (\forall V2m \in (ty_2Enum_2Enum^{A_27a}). (\forall V3e \in \\
& \quad A_27a. (\forall V4y \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V0x) \\
& \quad V1s)) \wedge (p (ap (ap (c_2Eprim_rec_2E_3C (ap V2m V3e)) (ap V2m V0x)))) \Rightarrow \\
& \quad ((p (ap (ap (ap (c_2Epred_set_2Eis_measure_maximal A_27a) \\
& \quad V2m) (ap (ap (c_2Epred_set_2EINSERT A_27a) V3e) V1s)) V4y)) \Leftrightarrow (\\
& \quad p (ap (ap (ap (c_2Epred_set_2Eis_measure_maximal A_27a) V2m) \\
& \quad V1s) V4y))))))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap (c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& \quad (ap c_2Enum_2ESUC V0n))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\
& \quad A_27a). (\forall V1y \in (ty_2Elist_2Elist A_27a). ((p (ap (ap (c_2Elist_2EisPREFIX \\
& \quad A_27a) V0x) V1y)) \wedge ((ap (c_2Elist_2ELENGTH A_27a) V0x) = (ap (c_2Elist_2ELENGTH \\
& \quad A_27a) V1y))) \Leftrightarrow (V0x = V1y))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Elist_2Elist A_27a)}). \\
& \quad ((ap (c_2Erich_list_2Ecommon_prefixes A_27a) V0s) = (ap (c_2Epred_set_2EBIGINTER \\
& \quad (ty_2Elist_2Elist A_27a) (ap (ap (c_2Epred_set_2EIMAGE (ty_2Elist_2Elist \\
& \quad A_27a) (2^{(ty_2Elist_2Elist A_27a)})) (\lambda V1l \in (ty_2Elist_2Elist \\
& \quad A_27a). (ap (c_2Epred_set_2EGSPEC (ty_2Elist_2Elist A_27a) \\
& \quad (ty_2Elist_2Elist A_27a)) (\lambda V2p \in (ty_2Elist_2Elist A_27a). \\
& \quad (ap (ap (c_2Epair_2E_2C (ty_2Elist_2Elist A_27a) 2) V2p) (ap (\\
& \quad ap (c_2Elist_2EisPREFIX A_27a) V2p) V1l)))))) V0s))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Elist_2Elist \\ & \quad A_27a).(p\ (ap\ (c_2Epred_set_2EFINITE\ (ty_2Elist_2Elist\ A_27a)) \\ & (ap\ (c_2Epred_set_2EGSPEC\ (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist \\ & \quad A_27a))\ (\lambda V1a \in (ty_2Elist_2Elist\ A_27a).(ap\ (ap\ (c_2Epair_2E_2C \\ & \quad (ty_2Elist_2Elist\ A_27a)\ 2)\ V1a)\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\ & \quad \quad A_27a)\ V1a)\ V0b)))))) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Elist_2Elist\ A_27a)}), \\ & (\forall V1p1 \in (ty_2Elist_2Elist\ A_27a).(\forall V2p2 \in (ty_2Elist_2Elist \\ & \quad A_27a).(((\neg(V0s = (c_2Epred_set_2EEMPTY\ (ty_2Elist_2Elist \\ & \quad A_27a)))) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Elist_2Elist\ A_27a)) \\ & \quad \quad V1p1)\ (ap\ (c_2Erich_list_2Ecommon_prefixes\ A_27a)\ V0s))) \wedge \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Elist_2Elist\ A_27a))\ V2p2)\ (ap\ (\\ & \quad \quad c_2Erich_list_2Ecommon_prefixes\ A_27a)\ V0s)))))) \Rightarrow ((p\ (ap\ (\\ & \quad ap\ (c_2Elist_2EisPREFIX\ A_27a)\ V1p1)\ V2p2)) \vee (p\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\ & \quad \quad A_27a)\ V2p2)\ V1p1)))))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Elist_2Elist\ A_27a)}), \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Elist_2Elist\ A_27a))\ (c_2Elist_2ENIL \\ & \quad A_27a))\ V0s)) \Rightarrow ((ap\ (c_2Erich_list_2Ecommon_prefixes\ A_27a) \\ & \quad V0s) = (ap\ (ap\ (c_2Epred_set_2EINSERT\ (ty_2Elist_2Elist\ A_27a)) \\ & \quad (c_2Elist_2ENIL\ A_27a))\ (c_2Epred_set_2EEMPTY\ (ty_2Elist_2Elist \\ & \quad \quad A_27a)))))) \end{aligned} \quad (90)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Elist_2Elist\ A_27a)}), \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Elist_2Elist\ A_27a))\ (c_2Elist_2ENIL \\ & \quad A_27a))\ V0s)) \Rightarrow ((ap\ (c_2Erich_list_2Elongest_prefix\ A_27a) \\ & \quad V0s) = (c_2Elist_2ENIL\ A_27a)))) \end{aligned} \quad (91)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (92)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (93)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (94)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (95)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (96)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (97)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (98)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (99)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (100)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (101)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (102)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (103)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (104)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (105)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (106)$$

Theorem 1

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0ys \in (ty_2Elist_2Elist \\ & \quad A_27a). (\forall V1xs \in (ty_2Elist_2Elist A_27a). (\forall V2x \in \\ & \quad A_27a. (\forall V3y \in A_27a. (((ap (c_2Erich_list_2Elongest_prefix \\ & \quad A_27a) (ap (ap (c_2Epred_set_2EINSERT (ty_2Elist_2Elist A_27a)) \\ & \quad (c_2Elist_2ENIL A_27a)) (ap (ap (c_2Epred_set_2EINSERT (ty_2Elist_2Elist \\ & \quad A_27a)) V0ys) (c_2Epred_set_2EEMPTY (ty_2Elist_2Elist A_27a)))))) = \\ & \quad (c_2Elist_2ENIL A_27a) \wedge (((ap (c_2Erich_list_2Elongest_prefix \\ & \quad A_27a) (ap (ap (c_2Epred_set_2EINSERT (ty_2Elist_2Elist A_27a)) \\ & \quad V1xs) (ap (ap (c_2Epred_set_2EINSERT (ty_2Elist_2Elist A_27a)) \\ & \quad (c_2Elist_2ENIL A_27a)) (c_2Epred_set_2EEMPTY (ty_2Elist_2Elist \\ & \quad A_27a)))))) = (c_2Elist_2ENIL A_27a) \wedge ((ap (c_2Erich_list_2Elongest_prefix \\ & \quad A_27a) (ap (ap (c_2Epred_set_2EINSERT (ty_2Elist_2Elist A_27a)) \\ & \quad (ap (ap (c_2Elist_2ECONS A_27a) V2x) V1xs)) (ap (ap (c_2Epred_set_2EINSERT \\ & \quad (ty_2Elist_2Elist A_27a)) (ap (ap (c_2Elist_2ECONS A_27a) V3y) \\ & \quad V0ys)) (c_2Epred_set_2EEMPTY (ty_2Elist_2Elist A_27a)))))) = \\ & \quad (ap (ap (ap (c_2Ebool_2ECOND (ty_2Elist_2Elist A_27a)) (ap (ap \\ & \quad (c_2Emin_2E_3D A_27a) V2x) V3y)) (ap (ap (c_2Elist_2ECONS A_27a) \\ & \quad V2x) (ap (c_2Erich_list_2Elongest_prefix A_27a) (ap (ap (c_2Epred_set_2EINSERT \\ & \quad (ty_2Elist_2Elist A_27a)) V1xs) (ap (ap (c_2Epred_set_2EINSERT \\ & \quad (ty_2Elist_2Elist A_27a)) V0ys) (c_2Epred_set_2EEMPTY (ty_2Elist_2Elist \\ & \quad A_27a)))))))) (c_2Elist_2ENIL A_27a))))))))) \end{aligned}$$