

thm_2Erich_list_2Elongest_prefix_SING (TM-SRD9hpQYcqrqkTo61uUdD1Hci8AWXCSr1)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}})_{A_27b})^{(ty_2Elist_2Elist\ A_27a)}) \quad (2)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})_{A_27a}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (4)$$

Definition 3 We define $c_2Ebool_2E_2EIN$ to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.\lambda V1f \in (2^{A-27a}).(ap\ V1f\ V0x))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_2E21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ P)\ V0P))$

Definition 6 We define $c_2Ebool_2E_2E5C_2E2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_2E21\ 2)\ V1t2))\ V0t1)$

Definition 7 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_2E21\ 2)\ V1t2))\ V0t1)$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b}})^{A_27a}) \end{aligned} \quad (5)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (6)$$

Definition 9 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2E$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$).

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (7)$$

Definition 12 We define $c_2Ebool_2E_F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 18 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_s$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EisPREFIX\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (11)$$

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 20 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E21\ 2))$

Definition 21 We define $c_2Erich_list_2Ecommon_prefixes$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Elist_2Elist\ A_27a)})$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (12)$$

Definition 22 We define $c_2Epred_set_2Eis_measure_maximal$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Enum_2Enum)$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (13)$$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 24 We define $c_2Erich_list_2Elongest_prefix$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Elist_2Elist\ A_27a)})$.

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ V1m)) \wedge (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ V0n))) \Rightarrow (V0n = V1m)))) \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (22)
\end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (23)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t)))))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\
& (2^{A_27a}).(((\exists V2x \in A_27a.(p \ (ap \ V0P \ V2x))) \wedge (\forall V3x \in \\
& A_27a.((p \ (ap \ V0P \ V3x)) \Rightarrow (p \ (ap \ V1Q \ V3x)))) \Rightarrow (p \ (ap \ V1Q \ (ap \ (c_2Emin_2E.40 \\
& A_27a) \ V0P)))))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (\\
& (p \ V1B) \wedge (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \quad (27)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A))))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (30)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.((((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.((((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c.2Ebool_2ECOND A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c.2Ebool_2ECOND A_{.27a}) V1Q) V3x_{.27} \\ & V5y_{.27})))))) \end{aligned} \quad (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ & A_{.27a}.((ap (ap (ap (c.2Ebool_2ECOND A_{.27a}) c.2Ebool_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \\ & (ap (ap (c.2Ebool_2ECOND A_{.27a}) c.2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\
& 2. (((\exists V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \Rightarrow (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\
& A_27a. ((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))) \wedge (((\exists V4x \in A_27a. (p\ (\\
& ap\ V0P\ V4x)) \wedge (p\ V1Q)) \Leftrightarrow (\exists V5x \in A_27a. ((p\ (ap\ V0P\ V5x)) \wedge (p \\
& V1Q)))) \wedge (((p\ V1Q) \wedge (\exists V6x \in A_27a. (p\ (ap\ V0P\ V6x)))) \Leftrightarrow (\exists V7x \in \\
& A_27a. ((p\ V1Q) \wedge (p\ (ap\ V0P\ V7x)))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A_27a). ((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (c_2Elist_2ENIL \\
& A_27a))\ V0l)) \Leftrightarrow True)) \wedge (\forall V1h \in A_27a. (\forall V2t \in (ty_2Elist_2Elist \\
& A_27a). (\forall V3l \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\
& A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1h)\ V2t))\ V3l)) \Leftrightarrow (p\ (ap\ (\\
& ap\ (ap\ (c_2Elist_2Elist_CASE\ A_27a\ 2)\ V3l)\ c_2Ebool_2EF)\ (\lambda V4h_27 \in \\
& A_27a. (\lambda V5t_27 \in (ty_2Elist_2Elist\ A_27a). (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V1h)\ V4h_27))\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\
& A_27a)\ V2t)\ V5t_27))))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\
& A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\
& A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\
& A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p\ (ap\ (ap \\
(c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((V0s = \\ (c_2Epred_set_2EEMPTY\ A_27a)) \vee (\exists V1x \in A_27a.(\exists V2t \in \\ (2^{A_27a}).((V0s = (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1x) \\ V2t)) \wedge (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ V2t)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1s \in \\ (2^{A_27a}).(\neg((ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V0x)\ V1s) = \\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0y \in A_27b.(\forall V1s \in (2^{A_27a}).(\forall V2f \in (A_27b^{A_27a}). \\ ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a.((V0y = (ap\ V2f\ V3x)) \wedge \\ (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}).((\\ \exists V1s \in (2^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V1s) \\ V0P))) \wedge (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)))) \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE \\ A_27a)\ (ap\ (c_2Epred_set_2EBIGINITER\ A_27a)\ V0P)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in (ty_2Enum_2Enum^{A_27a}). \\ (\forall V1s \in (2^{A_27a}).(((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\ V1s)) \wedge (\neg(V1s = (c_2Epred_set_2EEMPTY\ A_27a)))) \Rightarrow (\exists V2x \in \\ A_27a.(p\ (ap\ (ap\ (ap\ (c_2Epred_set_2Eis_measure_maximal\ A_27a) \\ V0m)\ V1s)\ V2x)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\ A_27a).(\forall V1y \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\ A_27a)\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ (c_2Elist_2ELENGTH \\ A_27a)\ V0x))\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1y)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\ A_27a).(\forall V1y \in (ty_2Elist_2Elist\ A_27a).(((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\ A_27a)\ V0x)\ V1y)) \wedge ((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0x) = (ap\ (c_2Elist_2ELENGTH \\ A_27a)\ V1y))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Elist_2Elist } A_27a)}). \\
& ((ap (c_2Erich_list_2Ecommon_prefixes } A_27a) V0s) = (ap (c_2Epred_set_2EBIGINTER \\
& (ty_2Elist_2Elist } A_27a)) (ap (ap (c_2Epred_set_2EIMAGE (ty_2Elist_2Elist \\
& A_27a) (2^{(ty_2Elist_2Elist } A_27a)})) (\lambda V1l \in (ty_2Elist_2Elist \\
& A_27a).(ap (c_2Epred_set_2EGSPEC (ty_2Elist_2Elist } A_27a) \\
& (ty_2Elist_2Elist } A_27a)) (\lambda V2p \in (ty_2Elist_2Elist } A_27a). \\
& (ap (ap (c_2Epair_2E_2C (ty_2Elist_2Elist } A_27a) 2) V2p) (ap (\\
& ap (c_2Elist_2EisPREFIX } A_27a) V2p) V1l)))))) V0s)))) \\
& \tag{49}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0b \in (ty_2Elist_2Elist \\
& A_27a).(p (ap (c_2Epred_set_2EFINITE (ty_2Elist_2Elist } A_27a)) \\
& (ap (c_2Epred_set_2EGSPEC (ty_2Elist_2Elist } A_27a) (ty_2Elist_2Elist \\
& A_27a)) (\lambda V1a \in (ty_2Elist_2Elist } A_27a).(ap (ap (c_2Epair_2E_2C \\
& (ty_2Elist_2Elist } A_27a) 2) V1a) (ap (ap (c_2Elist_2EisPREFIX \\
& A_27a) V1a) V0b)))))) \\
& \tag{50}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \\
& \tag{53}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \\
& \tag{54}
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \\
& \tag{56}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{65}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0s \in (ty_2Elist_2Elist \\
& A_{27a}). ((ap (c_2Erich_list_2Elongest_prefix A_{27a}) (ap (ap \\
& (c_2Epred_set_2INSERT (ty_2Elist_2Elist A_{27a}) V0s) (c_2Epred_set_2EMPTY \\
& (ty_2Elist_2Elist A_{27a})))) = V0s))
\end{aligned}$$