

thm_2Erich__list_2Emap__replicate (TMKy- oWQdZnkGJx5uwuBLguFNdxNjmJ1DMYF)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_7E` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_7E` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V 0t \in 2.V 0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V 0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V 0t)) \text{c_2Ebool_2E_7E}))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A 0) \quad (1)$$

Let `c_2Elist_2EMAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Elist_2EMAP } A. 27a \ A. 27b \in (((\text{ty_2Elist_2Elist } A. 27b)^{(\text{ty_2Elist_2Elist } A. 27a)})^{(A. 27b^{A-27a})}) \quad (2)$$

Let `c_2Elist_2ECONS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Elist_2ECONS } A. 27a \in (((\text{ty_2Elist_2Elist } A. 27a)^{(\text{ty_2Elist_2Elist } A. 27a)})^{A. 27a}) \quad (3)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \quad (4)$$

Let `c_2Enum_2EREP_num` : ι be given. Assume the following.

$$\text{c_2Enum_2EREP_num} \in (\text{omega}^{\text{ty_2Enum_2Enum}}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega^{\omega}}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega^{\omega}}) \quad (7)$$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (8)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Erich_list_2EREPLICATE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2EREPLICATE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{A_27a})^{ty_2Enum_2Enum} \quad (10)$$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ t2)\ t1))$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg \\ & (p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t)))))) \\
& \hspace{15em} (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a0 \in A_27a.(\forall V1a1 \in \\
& (ty_2Elist_2Elist\ A_27a).(\forall V2a0_27 \in A_27a.(\forall V3a1_27 \in \\
& (ty_2Elist_2Elist\ A_27a).(((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0a0) \\
& V1a1) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0_27)\ V3a1_27)) \Leftrightarrow ((V0a0 = \\
& V2a0_27) \wedge (V1a1 = V3a1_27)))))) \\
& \hspace{15em} (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))) \\
& \hspace{15em} (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a.((ap\ (ap\ (c_2Erich_list_2EREPLICATE \\
& A_27a)\ c_2Enum_2E0)\ V0x) = (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1n \in \\
& ty_2Enum_2Enum.(\forall V2x \in A_27a.((ap\ (ap\ (c_2Erich_list_2EREPLICATE \\
& A_27a)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2x) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a) \\
& V2x)\ (ap\ (ap\ (c_2Erich_list_2EREPLICATE\ A_27a)\ V1n)\ V2x)))))) \\
& \hspace{15em} (18)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}).(\forall V1n \in ty_2Enum_2Enum.(\forall V2x \in \\
& A_27a.((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V0f)\ (ap\ (ap\ (c_2Erich_list_2EREPLICATE \\
& A_27a)\ V1n)\ V2x)) = (ap\ (ap\ (c_2Erich_list_2EREPLICATE\ A_27b) \\
& V1n)\ (ap\ V0f\ V2x))))))
\end{aligned}$$