thm_2EringNorm_2Ecanonical__sum__merge__def (TMSZ2dudFpqjNeXvRuT6XYnzxDmBSyVrpga)

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Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \tag{1}$$

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \tag{2}$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{3}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli} \tag{5}$$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{6}$$

Let $c\_2Equote\_2Eindex\_\_compare : \iota$ be given. Assume the following.

$$c\_2Equote\_2Eindex\_\_compare \in ((ty\_2EternaryComparisons\_2Eordering^{ty\_2Equote\_2Eindex})^{ty\_2Equote\_2E} \tag{7}$$

Let $c\_2EternaryComparisons\_2Elist\_\_compare : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EternaryComparisons\ A\_27a\ A\_27b \in (((ty\_2EternaryComparisons\_2Eordering^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((ty} \tag{8}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{9}$$

Let $c\_2EternaryComparisons\_2Eordering2num : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2Eordering2num \in (ty\_2Enum\_2Enum^{ty\_2EternaryComparisons\_2Eordering}) \tag{10}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{11}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{12}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{14}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 16 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 17 *We define* c_2EternaryComparisons_2Eordering__CASE *to be* $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Eternary$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \quad (16)$$

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \quad (17)$$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (18)$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (19)$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (20)$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (21)$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \quad (22)$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \quad (23)$$

Definition 18 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(a$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_s$$
$$A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \quad (24)$$

Definition 19 *We define* c_2EringNorm_2Er__canonical__sum__merge *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2I$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \quad (25)$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \quad (26)$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \quad (27)$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum \\
A\_27a).(\forall V1t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a). \\
(\forall V2sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V3l2 \in \\
(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4l1 \in (ty\_2Elist\_2Elist \\
ty\_2Equote\_2Eindex).(\forall V5c2 \in A\_27a.(\forall V6c1 \in A\_27a. \\
((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V2sr) \\
(ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1)\ V4l1)\ V1t1)) \\
(ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V5c2)\ V3l2)\ V0t2)) = \\
(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum \\
A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare \\
ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare) \\
V4l1)\ V3l2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1) \\
V4l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a) \\
V2sr)\ V1t1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V5c2) \\
V3l2)\ V0t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a) \\
(ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V2sr)\ V6c1) \\
V5c2))\ V4l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge \\
A\_27a)\ V2sr)\ V1t1)\ V0t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom \\
A\_27a)\ V5c2)\ V3l2)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge \\
A\_27a)\ V2sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1) \\
V4l1)\ V1t1))\ V0t2)))))))))))) \wedge ((\forall V7t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum \\
A\_27a).(\forall V8t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a). \\
(\forall V9sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V10l2 \in \\
(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V11l1 \in (ty\_2Elist\_2Elist \\
ty\_2Equote\_2Eindex).(\forall V12c1 \in A\_27a.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge \\
A\_27a)\ V9sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V12c1) \\
V11l1)\ V8t1))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V10l2) \\
V7t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE} \\
(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare \\
ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare) \\
V11l1)\ V10l2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a) \\
V12c1)\ V11l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge \\
A\_27a)\ V9sr)\ V8t1)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a) \\
V10l2)\ V7t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a) \\
(ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V9sr)\ V12c1) \\
(ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V9sr)))\ V11l1) \\
(ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V9sr) \\
V8t1)\ V7t2)))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V10l2) \\
(ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V9sr) \\
(ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V12c1)\ V11l1) \\
V8t1))\ V7t2)))))))))) \wedge ((\forall V13t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum \\
A\_27a).(\forall V14t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a). \\
(\forall V15sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V16l2 \in \\
(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V17l1 \in (ty\_2Elist\_2Elist \\
ty\_2Equote\_2Eindex).(\forall V18c2 \in A\_27a.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge \\
A\_27a)\ V15sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V17l1) \\
V14t1))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V18c2) \\
V16l2)\ V13t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE} \\
(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare \\
ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare) \\
V17l1)\ V16l2))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a) \\
V17l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a) \\
V15sr)\ V14t1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a) \\
V18c2)\ V16l2)\ V13t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom \\
A\_27a)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V15sr) \\
(ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V15sr))\ V18c2)) \\
V17l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a) \\
V15sr)\ V14t1)\ V13t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom \\
A\_27a)\ V18c2)\ V16l2)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge
\end{gathered}
$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in A\_27a.(\forall V1a0 \in A\_27a.(\forall V2f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V3f0 \in ((A\_27a^{A\_27a})^{A\_27a}). ((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V0a)\ V1a0)\ V2f)\ V3f0)) = V0a)))))\wedge((\forall V4a \in A\_27a.(\forall V5a0 \in A\_27a. (\forall V6f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V7f0 \in ((A\_27a^{A\_27a})^{A\_27a}). ((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V4a)\ V5a0)\ V6f)\ V7f0)) = V5a0)))))\wedge((\forall V8a \in A\_27a.(\forall V9a0 \in A\_27a. (\forall V10f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V11f0 \in ((A\_27a^{A\_27a})^{A\_27a}). ((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V8a)\ V9a0)\ V10f)\ V11f0)) = V10f)))))\wedge(\forall V12a \in A\_27a.(\forall V13a0 \in A\_27a.(\forall V14f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V15f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0)))))))) \tag{31}$$

Theorem 1

$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering$
$A\_27a).((\forall V1t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).$
$(\forall V2t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V3l2 \in$
$(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4l1 \in (ty\_2Elist\_2Elist$
$ty\_2Equote\_2Eindex).(\forall V5c2 \in A\_27a.(\forall V6c1 \in A\_27a.$
$((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}\ A\_27a)$
$V0r)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1)\ V4l1)$
$V2t1))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V5c2)\ V3l2)$
$V1t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}$
$(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare$
$ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)$
$V4l1)\ V3l2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1)$
$V4l1)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}\ A\_27a)$
$V0r)\ V2t1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V5c2)$
$V3l2)\ V1t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$
$(ap\ (ap\ (ap\ (c\_2Ering\_2Ering\_\_RP\ A\_27a)\ V0r)\ V6c1)\ V5c2))\ V4l1)$
$(ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}\ A\_27a)$
$V0r)\ V2t1)\ V1t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$
$V5c2)\ V3l2)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}$
$A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c1)$
$V4l1)\ V2t1))\ V1t2))))))))))\wedge((\forall V7t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum$
$A\_27a).(\forall V8t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).$
$(\forall V9l2 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V10l1 \in$
$(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V11c1 \in A\_27a.$
$((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}\ A\_27a)$
$V0r)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V11c1)\ V10l1)$
$V8t1))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V9l2)\ V7t2)) =$
$(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum$
$A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare$
$ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)$
$V10l1)\ V9l2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$
$V11c1)\ V10l1)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}$
$A\_27a)\ V0r)\ V8t1)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)$
$V9l2)\ V7t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$
$(ap\ (ap\ (ap\ (c\_2Ering\_2Ering\_\_RP\ A\_27a)\ V0r)\ V11c1)\ (ap\ (c\_2Ering\_2Ering\_\_R1$
$A\_27a)\ V0r)))\ V10l1)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}$
$A\_27a)\ V0r)\ V8t1)\ V7t2)))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist$
$A\_27a)\ V9l2)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}$
$A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V11c1)$
$V10l1)\ V8t1))\ V7t2)))))))))\wedge((\forall V12t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum$
$A\_27a).(\forall V13t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).$
$(\forall V14l2 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V15l1 \in$
$(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V16c2 \in A\_27a.$
$((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}\ A\_27a)$
$V0r)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V15l1)\ V13t1))$
$(ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V16c2)\ V14l2)$
$V12t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}$
$(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare$
$ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)$
$V15l1)\ V14l2))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)$
$V15l1)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}$
$A\_27a)\ V0r)\ V13t1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)$
$V16c2)\ V14l2)\ V12t2))))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom$
$A\_27a)\ (ap\ (ap\ (ap\ (c\_2Ering\_2Ering\_\_RP\ A\_27a)\ V0r)\ (ap\ (c\_2Ering\_2Ering\_\_R1$
$A\_27a)\ V0r))\ V16c2))\ V15l1)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}$
$A\_27a)\ V0r)\ V13t1)\ V12t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom$
$A\_27a)\ V16c2)\ V14l2)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}$
$A\_27a)\ V0r)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V15l1)$
$V13t1))\ V12t2)))))))))\wedge((\forall V17t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum$