thm_2EringNorm_2Ecanonical__sum__prod__def (TMT1uwf7a6Bho31o9p9NZoFg78pyKnX4REf)

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Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \tag{1}$$

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \tag{2}$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{3}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli} \tag{5}$$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \tag{6}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{7}$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \tag{8}$$

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \tag{9}$$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{10}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{11}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{12}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{13}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{14}$$

Definition 6 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Definition 7 *We define* c_2EringNorm_2Er__canonical__sum__prod *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Eri$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_$$
$$A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2E} \quad (15)$$

Definition 8 *We define* c_2EringNorm_2Er__canonical__sum__scalar3 *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2E$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_sur$$
$$A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2El} \quad (16)$$

Definition 9 *We define* c_2EringNorm_2Er__canonical__sum__scalar2 *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2E$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_s$$
$$A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \quad (17)$$

Definition 10 *We define* c_2EringNorm_2Er__canonical__sum__merge *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2I$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1c1 \in A\_27a.(\forall V2l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V4s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V0sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c1)\ V2l1)\ V3t1))\ V4s2) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V0sr)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V0sr)\ V1c1)\ V2l1)\ V4s2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V0sr)\ V3t1)\ V4s2)))))))) \wedge ((\forall V5sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V6l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V7t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V8s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V5sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V6l1)\ V7t1))\ V8s2) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V5sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V5sr)\ V6l1)\ V8s2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V5sr)\ V7t1)\ V8s2))))))) \wedge (\forall V9sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V10s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V9sr)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))\ V10s2) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))) \tag{19}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering\ A\_27a).((\forall V1c1 \in A\_27a.(\forall V2l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V4s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_prod}\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c1)\ V2l1)\ V3t1))\ V4s2) = (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar3}\ A\_27a)\ V0r)\ V1c1)\ V2l1)\ V4s2))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_prod}\ A\_27a)\ V0r)\ V3t1)\ V4s2))))))) \land ((\forall V5l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V6t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).(\forall V7s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_prod}\ A\_27a)\ V0r)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V5l1)\ V6t1))\ V7s2) = (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_merge}\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar2}\ A\_27a)\ V0r)\ V5l1)\ V7s2))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_prod}\ A\_27a)\ V0r)\ V6t1)\ V7s2)))))) \land (\forall V8s2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_prod}\ A\_27a)\ V0r)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))\ V8s2) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))))$$