thm_2EringNorm_2Ecanonical__sum__scalar2__def (TMUgAi5vAHcWU1EdLrRC2n8y62pqw8Yf7cq)

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Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \tag{1}$$

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \tag{2}$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{3}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli \tag{5}$$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{6}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{7}$$

Let $c\_2Equote\_2Eindex\_\_compare : \iota$ be given. Assume the following.

$$c\_2Equote\_2Eindex\_\_compare \in ((ty\_2EternaryComparisons\_2Eordering^{ty\_2Equote\_2Eindex})^{ty\_2Equote\_2E \tag{8}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Equote_2Eindex__lt *to be* $\lambda V0i1 \in ty\_2Equote\_2Eindex.\lambda V1i2 \in ty\_2Equote\_2E$

Let $c\_2EternaryComparisons\_2Elist\_\_merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a \in ((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((2^{A\_27a})^{A\_27a})}) \tag{9}$$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \tag{10}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{11}$$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_su\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2El} \tag{12}$$

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \tag{13}$$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{14}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{15}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{16}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{17}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{18}$$

Definition 7 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Definition 8 *We define* c_2EringNorm_2Er__canonical__sum__scalar2 *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2E$

Let $c\_2Ecanonical\_2Evarlist\_\_insert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Evarlist\_\_i\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2El} \tag{19}$$

Definition 9 *We define* c_2EringNorm_2Er__varlist__insert *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27$

Let $c\_2Ecanonical\_2Emonom\_\_insert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Emonon\ A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2E} \tag{20}$$

Definition 10 *We define* c_2EringNorm_2Er__monom__insert *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{21}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V2c \in A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V0sr)\ V1l0)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V2c)\ V3l)\ V4t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Emonom\_\_insert\ A\_27a)\ V0sr)\ V2c)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V1l0)\ V3l))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V0sr)\ V1l0)\ V4t))))))))\wedge((\forall V5sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V6l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V7l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V8t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V5sr)\ V6l0)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V7l)\ V8t)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Evarlist\_\_insert\ A\_27a)\ V5sr)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V6l0)\ V7l))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V5sr)\ V6l0)\ V8t)))))))\wedge(\forall V9sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V10l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2\ A\_27a)\ V9sr)\ V10l0)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))) \tag{22}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering\ A\_27a).((\forall V1l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V2c \in A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar2}\ A\_27a)\ V0r)\ V1l0)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V2c)\ V3l)\ V4t)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_monom\_\_insert}\ A\_27a)\ V0r)\ V2c)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V1l0)\ V3l))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar2}\ A\_27a)\ V0r)\ V1l0)\ V4t)))))))\wedge((\forall V5l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V6l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V7t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar2}\ A\_27a)\ V0r)\ V5l0)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V6l)\ V7t)) = (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_varlist\_\_insert}\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V5l0)\ V6l))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar2}\ A\_27a)\ V0r)\ V5l0)\ V7t))))))\wedge(\forall V8l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar2}\ A\_27a)\ V0r)\ V8l0)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))))$$