thm_2EringNorm_2Ecanonical__sum__scalar3__def (TMJ9QLbuS5frJg2QWyphrHzVDq45ssrL9qy)

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Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \tag{1}$$

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \tag{2}$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{3}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli} \tag{5}$$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{6}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{7}$$

Let $c\_2Equote\_2Eindex\_\_compare : \iota$ be given. Assume the following.

$$c\_2Equote\_2Eindex\_\_compare \in ((ty\_2EternaryComparisons\_2Eordering^{ty\_2Equote\_2Eindex})^{ty\_2Equote\_2E} \tag{8}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Equote_2Eindex__lt *to be* $\lambda V0i1 \in ty\_2Equote\_2Eindex.\lambda V1i2 \in ty\_2Equote\_2E$

Let $c\_2EternaryComparisons\_2Elist\_\_merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a \in ((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((2^{A\_27a})^{A\_27a})}) \tag{9}$$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \tag{10}$$

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \tag{11}$$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{12}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{13}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{14}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{15}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{16}$$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_ A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2E} \tag{17}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{18}$$

Definition 5 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Definition 6 *We define* c_2EringNorm_2Er__canonical__sum__scalar3 *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2E$

Let $c\_2Ecanonical\_2Emonom\_\_insert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Emonor$$
$$A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2E} \tag{19}$$

Definition 7 *We define* c_2EringNorm_2Er__monom__insert *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{20}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{21}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{22}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{23}$$

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1c0 \in A\_27a.(\forall V2l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3c \in A\_27a.(\forall V4l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V5t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V0sr)\ V1c0)\ V2l0)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V3c)\ V4l)\ V5t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Emonom\_\_insert\ A\_27a)\ V0sr)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V0sr)\ V1c0)\ V3c))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V2l0)\ V4l))\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V0sr)\ V1c0)\ V2l0)\ V5t))))))))) \wedge ((\forall V6sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V7c0 \in A\_27a.(\forall V8l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V9l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V10t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V6sr)\ V7c0)\ V8l0)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V9l)\ V10t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Emonom\_\_insert\ A\_27a)\ V6sr)\ V7c0)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V8l0)\ V9l))\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V6sr)\ V7c0)\ V8l0)\ V10t)))))))) \wedge (\forall V11sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V12c0 \in A\_27a.(\forall V13l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3\ A\_27a)\ V11sr)\ V12c0)\ V13l0)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in A\_27a.(\forall V1a0 \in A\_27a.(\forall V2f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V3f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V0a)\ V1a0)\ V2f)\ V3f0)) = V0a))))) \wedge ((\forall V4a \in A\_27a.(\forall V5a0 \in A\_27a.(\forall V6f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V7f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V4a)\ V5a0)\ V6f)\ V7f0)) = V5a0))))) \wedge ((\forall V8a \in A\_27a.(\forall V9a0 \in A\_27a.(\forall V10f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V11f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V8a)\ V9a0)\ V10f)\ V11f0)) = V10f))))) \wedge (\forall V12a \in A\_27a.(\forall V13a0 \in A\_27a.(\forall V14f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V15f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0)))))))) \tag{26}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering\ A\_27a).((\forall V1c0 \in A\_27a.(\forall V2l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3c \in A\_27a.(\forall V4l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V5t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ \mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar3}\ A\_27a)\ V0r)\ V1c0)\ V2l0)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V3c)\ V4l)\ V5t)) = (ap\ (ap\ (ap\ (ap\ \mathsf{c\_2EringNorm\_2Er\_\_monom\_\_insert}\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (c\_2Ering\_2Ering\_\_RM\ A\_27a)\ V0r)\ V1c0)\ V3c))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V2l0)\ V4l))\ (ap\ (ap\ (ap\ (ap\ \mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar3}\ A\_27a)\ V0r)\ V1c0)\ V2l0)\ V5t)))))))) \wedge ((\forall V6c0 \in A\_27a.(\forall V7l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V8l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V9t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ \mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar3}\ A\_27a)\ V0r)\ V6c0)\ V7l0)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V8l)\ V9t)) = (ap\ (ap\ (ap\ (ap\ \mathsf{c\_2EringNorm\_2Er\_\_monom\_\_insert}\ A\_27a)\ V0r)\ V6c0)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ ty\_2Equote\_2Eindex)\ \mathsf{c\_2Equote\_2Eindex\_\_lt})\ V7l0)\ V8l))\ (ap\ (ap\ (ap\ (ap\ \mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar3}\ A\_27a)\ V0r)\ V6c0)\ V7l0)\ V9t)))))))) \wedge (\forall V10c0 \in A\_27a.(\forall V11l0 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (ap\ \mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_scalar3}\ A\_27a)\ V0r)\ V10c0)\ V11l0)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))))$$