thm_2EringNorm_2Ecanonical__sum__simplify__def (TMFa6r2St7o2wZTTjB1PXhrkaLHVMR5fxRf)

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Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \tag{1}$$

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \tag{2}$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{3}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli \tag{5}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \tag{6}$$

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \tag{7}$$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{8}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{9}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{10}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{11}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{12}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{13}$$

Definition 9 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Esemi\_\_ri \tag{14}$$

Definition 10 *We define* c_2EringNorm_2Er__canonical__sum__simplify *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{15}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{16}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{17}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1c \in A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V0sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c)\ V2l)\ V3t)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1c)\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ V0sr)))\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V0sr)\ V3t))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1c)\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V0sr)))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V2l)\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V0sr)\ V3t)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c)\ V2l)\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V0sr)\ V3t)))))))))) \wedge ((\forall V4sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V5l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V6t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V4sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V5l)\ V6t)) = (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V5l)\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V4sr)\ V6t)))))) \wedge (\forall V7sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a)\ V7sr)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in A\_27a.(\forall V1a0 \in A\_27a.(\forall V2f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V3f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V0a)\ V1a0)\ V2f)\ V3f0)) = V0a))))) \wedge ((\forall V4a \in A\_27a.(\forall V5a0 \in A\_27a.(\forall V6f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V7f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V4a)\ V5a0)\ V6f)\ V7f0)) = V5a0))))) \wedge ((\forall V8a \in A\_27a.(\forall V9a0 \in A\_27a.(\forall V10f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V11f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V8a)\ V9a0)\ V10f)\ V11f0)) = V10f))))) \wedge (\forall V12a \in A\_27a.(\forall V13a0 \in A\_27a.(\forall V14f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V15f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0)))))))) \tag{21}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering\ A\_27a).((\forall V1c \in A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_simplify}\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c)\ V2l)\ V3t)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1c)\ (ap\ (c\_2Ering\_2Ering\_\_R0\ A\_27a)\ V0r)))\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_simplify}\ A\_27a)\ V0r)\ V3t))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1c)\ (ap\ (c\_2Ering\_2Ering\_\_R1\ A\_27a)\ V0r)))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V2l)\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_simplify}\ A\_27a)\ V0r)\ V3t)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V1c)\ V2l)\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_simplify}\ A\_27a)\ V0r)\ V3t))))))))) \land ((\forall V4l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V5t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_simplify}\ A\_27a)\ V0r)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V4l)\ V5t)) = (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V4l)\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_simplify}\ A\_27a)\ V0r)\ V5t))))) \land ((ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_canonical\_\_sum\_\_simplify}\ A\_27a)\ V0r)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))$$