thm_2EringNorm_2Eics__aux__def (TMEoomgb6S8VuYEVQvbvLbg43ND45xGD5vs)

October 26, 2020

Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \tag{1}$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{2}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{3}$$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \tag{4}$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli} \tag{5}$$

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \tag{6}$$

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \tag{7}$$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{8}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{9}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{10}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{11}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{12}$$

Let $ty\_2Equote\_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Equote\_2Evarmap\ A0) \tag{13}$$

Let $c\_2Ecanonical\_2Eics\_\_aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Eics\_\_aux\ A\_27a \in ((((A\_27a^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_r} \tag{14}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{15}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Definition 5 *We define* c_2EringNorm_2Er__ics__aux *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(a$

Let $c\_2Ecanonical\_2Einterp\_\_m : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_m\ A\_27a \in ((((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring} \tag{16}$$

Definition 6 *We define* c_2EringNorm_2Er__interp__m *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).$

Let $c\_2Ecanonical\_2Einterp\_\_vl : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a \in (((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{17}$$

Definition 7 *We define* c_2EringNorm_2Er__interp__vl *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).($

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{18}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{19}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{20}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{21}$$

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2a \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V0sr)\ V1vm)\ V2a)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = V2a)))) \wedge ((\forall V3sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V4vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V5a \in A\_27a.(\forall V6l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V7t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V3sr)\ V4vm)\ V5a)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V6l)\ V7t)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V3sr)\ V5a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V3sr)\ V4vm)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V3sr)\ V4vm)\ V6l))\ V7t)))))))) \wedge (\forall V8sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V9vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V10a \in A\_27a.(\forall V11c \in A\_27a.(\forall V12l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V13t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V8sr)\ V9vm)\ V10a)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V11c)\ V12l)\ V13t)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V8sr)\ V10a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V8sr)\ V9vm)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V8sr)\ V9vm)\ V11c)\ V12l))\ V13t)))))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in A\_27a.(\forall V1a0 \in A\_27a.(\forall V2f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V3f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V0a)\ V1a0)\ V2f)\ V3f0)) = V0a))))) \wedge ((\forall V4a \in A\_27a.(\forall V5a0 \in A\_27a.(\forall V6f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V7f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V4a)\ V5a0)\ V6f)\ V7f0)) = V5a0))))) \wedge ((\forall V8a \in A\_27a.(\forall V9a0 \in A\_27a.(\forall V10f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V11f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V8a)\ V9a0)\ V10f)\ V11f0)) = V10f))))) \wedge (\forall V12a \in A\_27a.(\forall V13a0 \in A\_27a.(\forall V14f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V15f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0)))))))) \tag{24}$$

Theorem 1

$$\begin{gathered}
\forall A_27a.\mathit{nonempty}\ A_27a \Rightarrow (\forall V0r \in (\mathit{ty_2Ering_2Ering}\\
A_27a).((\forall V1vm \in (\mathit{ty_2Equote_2Evarmap}\ A_27a).(\forall V2a \in\\
A_27a.((\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c_2EringNorm_2Er__ics__aux}\ A_27a)\ V0r)\\
V1vm)\ V2a)\ (\mathit{c_2Ecanonical_2ENil__monom}\ A_27a)) = V2a)))\wedge((\forall V3vm \in\\
(\mathit{ty_2Equote_2Evarmap}\ A_27a).(\forall V4a \in A_27a.(\forall V5l \in\\
(\mathit{ty_2Elist_2Elist}\ \mathit{ty_2Equote_2Eindex}).(\forall V6t \in (\mathit{ty_2Ecanonical_2Ecanonical__sum}\\
A_27a).((\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c_2EringNorm_2Er__ics__aux}\ A_27a)\ V0r)\\
V3vm)\ V4a)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Ecanonical_2ECons__varlist}\ A_27a)\ V5l)\\
V6t)) = (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Ering_2Ering__RP}\ A_27a)\ V0r)\ V4a)\ (\mathit{ap}\ (\mathit{ap}\\
(\mathit{ap}\ (\mathit{ap}\ (\mathsf{c_2EringNorm_2Er__ics__aux}\ A_27a)\ V0r)\ V3vm)\ (\mathit{ap}\ (\mathit{ap}\ (\\
\mathit{ap}\ (\mathsf{c_2EringNorm_2Er__interp__vl}\ A_27a)\ V0r)\ V3vm)\ V5l))\ V6t)))))))\wedge\\
(\forall V7vm \in (\mathit{ty_2Equote_2Evarmap}\ A_27a).(\forall V8a \in A_27a.\\
(\forall V9c \in A_27a.(\forall V10l \in (\mathit{ty_2Elist_2Elist}\ \mathit{ty_2Equote_2Eindex}).\\
(\forall V11t \in (\mathit{ty_2Ecanonical_2Ecanonical__sum}\ A_27a).((\mathit{ap}\\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c_2EringNorm_2Er__ics__aux}\ A_27a)\ V0r)\ V7vm)\ V8a)\\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Ecanonical_2ECons__monom}\ A_27a)\ V9c)\ V10l)\ V11t)) =\\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c_2Ering_2Ering__RP}\ A_27a)\ V0r)\ V8a)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\\
(\mathsf{c_2EringNorm_2Er__ics__aux}\ A_27a)\ V0r)\ V7vm)\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\\
\mathsf{c_2EringNorm_2Er__interp__m}\ A_27a)\ V0r)\ V7vm)\ V9c)\ V10l))\ V11t))))))))))
\end{gathered}$$