thm_2EringNorm_2Einterp__cs__def (TMMpE5TFPftXYhzsTT6sqxnn1Hz2U9D4TW5)

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Let $\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\mathit{nonempty}\ A0 \Rightarrow \mathit{nonempty}\ (\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A0) \tag{1}$$

Let $\mathit{ty\_2Equote\_2Eindex} : \iota$ be given. Assume the following.

$$\mathit{nonempty}\ \mathit{ty\_2Equote\_2Eindex} \tag{2}$$

Let $\mathit{ty\_2Elist\_2Elist} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\mathit{nonempty}\ A0 \Rightarrow \mathit{nonempty}\ (\mathit{ty\_2Elist\_2Elist}\ A0) \tag{3}$$

Let $\mathit{c\_2Ecanonical\_2ECons\_\_monom} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \mathit{c\_2Ecanonical\_2ECons\_\_monom}\ A\_27a \in ((((\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a)^{(\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a)})^{(\mathit{ty\_2Elist\_2Elist}\ \mathit{ty\_2Eq} \tag{4}$$

Let $\mathit{c\_2Ecanonical\_2ECons\_\_varlist} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \mathit{c\_2Ecanonical\_2ECons\_\_varlist}\ A\_27a \in (((\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a)^{(\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a)})^{(\mathit{ty\_2Elist\_2Eli} \tag{5}$$

Let $\mathit{c\_2Ecanonical\_2ENil\_\_monom} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \mathit{c\_2Ecanonical\_2ENil\_\_monom}\ A\_27a \in (\mathit{ty\_2Ecanonical\_2Ecanonical\_\_sum}\ A\_27a) \tag{6}$$

Let $\mathit{ty\_2Ering\_2Ering} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\mathit{nonempty}\ A0 \Rightarrow \mathit{nonempty}\ (\mathit{ty\_2Ering\_2Ering}\ A0) \tag{7}$$

Let $\mathit{c\_2Ering\_2Ering\_\_RM} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \mathit{c\_2Ering\_2Ering\_\_RM}\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(\mathit{ty\_2Ering\_2Ering}\ A\_27a)}) \tag{8}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{9}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{10}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{11}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{12}$$

Let $ty\_2Equote\_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Equote\_2Evarmap\ A0) \tag{13}$$

Let $c\_2Ecanonical\_2Einterp\_\_cs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a \in (((A\_27a^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27} \tag{14}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{15}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Definition 5 *We define* c_2EringNorm_2Er__interp__cs *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).$

Let $c\_2Ecanonical\_2Eics\_\_aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Eics\_\_aux\ A\_27a \in ((((A\_27a^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_r} \tag{16}$$

Definition 6 *We define* c_2EringNorm_2Er__ics__aux *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(a$

Let $c\_2Ecanonical\_2Einterp\_\_m : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_m\ A\_27a \in ((((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring} \tag{17}$$

Definition 7 *We define* c_2EringNorm_2Er__interp__m *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).$

Let $c\_2Ecanonical\_2Einterp\_\_vl : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a \in (((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{18}$$

Definition 8 *We define* c_2EringNorm_2Er__interp__vl *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).($

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{19}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{20}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{21}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{22}$$

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a)\ V0sr)\ V1vm)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ V0sr)))) \wedge ((\forall V2sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V3vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V4l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V5t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a)\ V2sr)\ V3vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V4l)\ V5t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V2sr)\ V3vm)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V2sr)\ V3vm)\ V4l))\ V5t)))))) \wedge (\forall V6sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V7vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V8c \in A\_27a.(\forall V9l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V10t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a)\ V6sr)\ V7vm)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V8c)\ V9l)\ V10t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eics\_\_aux\ A\_27a)\ V6sr)\ V7vm)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V6sr)\ V7vm)\ V8c)\ V9l))\ V10t))))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in A\_27a.(\forall V1a0 \in A\_27a.(\forall V2f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V3f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V0a)\ V1a0)\ V2f)\ V3f0)) = V0a))))) \wedge ((\forall V4a \in A\_27a.(\forall V5a0 \in A\_27a.(\forall V6f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V7f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V4a)\ V5a0)\ V6f)\ V7f0)) = V5a0))))) \wedge ((\forall V8a \in A\_27a.(\forall V9a0 \in A\_27a.(\forall V10f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V11f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V8a)\ V9a0)\ V10f)\ V11f0)) = V10f))))) \wedge (\forall V12a \in A\_27a.(\forall V13a0 \in A\_27a.(\forall V14f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V15f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0)))))))) \tag{25}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering\ A\_27a).((\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_cs}\ A\_27a)\ V0r)\ V1vm)\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)) = (ap\ (c\_2Ering\_2Ering\_\_R0\ A\_27a)\ V0r))) \wedge ((\forall V2vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V3l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_cs}\ A\_27a)\ V0r)\ V2vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ V3l)\ V4t)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_ics\_\_aux}\ A\_27a)\ V0r)\ V2vm)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_vl}\ A\_27a)\ V0r)\ V2vm)\ V3l))\ V4t))))) \wedge (\forall V5vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V6c \in A\_27a.(\forall V7l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V8t \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_cs}\ A\_27a)\ V0r)\ V5vm)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V6c)\ V7l)\ V8t)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_ics\_\_aux}\ A\_27a)\ V0r)\ V5vm)\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_m}\ A\_27a)\ V0r)\ V5vm)\ V6c)\ V7l))\ V8t)))))))))$$