thm_2EringNorm_2Einterp__sp__def (TMXcz7EKqgroV6FHf225VCasRUxp4uQ3AoT)

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Let $ty\_2Ecanonical\_2Espolynom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Espolynom\ A0) \tag{1}$$

Let $c\_2Ecanonical\_2ESPmult : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPmult\ A\_27a \in (((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)}) \tag{2}$$

Let $c\_2Ecanonical\_2ESPplus : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPplus\ A\_27a \in (((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)}) \tag{3}$$

Let $ty\_2Equote\_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Equote\_2Evarmap\ A0) \tag{4}$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{5}$$

Let $c\_2Equote\_2Evarmap\_\_find : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Equote\_2Evarmap\_\_find\ A\_27a \in ((A\_27a^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{ty\_2Equote\_2Eindex}) \tag{6}$$

Let $c\_2Ecanonical\_2ESPvar : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPvar\ A\_27a \in ((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{ty\_2Equote\_2Eindex}) \tag{7}$$

Let $c\_2Ecanonical\_2ESPconst : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPconst\ A\_27a \in ((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{A\_27a}) \tag{8}$$

Let $ty\_2Ering\_2Ering : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Ering\_2Ering\ A0) \tag{9}$$

Let $c\_2Ering\_2Ering\_\_RM : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{10}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{11}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{12}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{13}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{14}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{15}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Let $c\_2Ecanonical\_2Einterp\_\_sp : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a \in (((A\_27a^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{16}$$

Definition 5 *We define* c_2EringNorm_2Er__interp__sp *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{17}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{18}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{19}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{20}$$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2c \in A\_27a.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V0sr)\ V1vm)\ (ap\ (c\_2Ecanonical\_2ESPconst\ A\_27a)\ V2c)) = V2c)))) \wedge ((\forall V3sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V4vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V5i \in ty\_2Equote\_2Eindex.((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V3sr)\ V4vm)\ (ap\ (c\_2Ecanonical\_2ESPvar\ A\_27a)\ V5i)) = (ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a)\ V5i)\ V4vm))))) \wedge ((\forall V6sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V7vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V8p1 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V9p2 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V6sr)\ V7vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPplus\ A\_27a)\ V8p1)\ V9p2)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V6sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V6sr)\ V7vm)\ V8p1))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V6sr)\ V7vm)\ V9p2))))))) \wedge (\forall V10sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V11vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V12p1 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V13p2 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V10sr)\ V11vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPmult\ A\_27a)\ V12p1)\ V13p2)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V10sr)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V10sr)\ V11vm)\ V12p1))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_sp\ A\_27a)\ V10sr)\ V11vm)\ V13p2)))))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in A\_27a.(\forall V1a0 \in A\_27a.(\forall V2f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V3f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V0a)\ V1a0)\ V2f)\ V3f0)) = V0a))))) \wedge ((\forall V4a \in A\_27a.(\forall V5a0 \in A\_27a.(\forall V6f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V7f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V4a)\ V5a0)\ V6f)\ V7f0)) = V5a0))))) \wedge ((\forall V8a \in A\_27a.(\forall V9a0 \in A\_27a.(\forall V10f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V11f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V8a)\ V9a0)\ V10f)\ V11f0)) = V10f))))) \wedge (\forall V12a \in A\_27a.(\forall V13a0 \in A\_27a.(\forall V14f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V15f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0)))))))) \tag{23}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering\ A\_27a).((\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2c \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_sp}\ A\_27a)\ V0r)\ V1vm)\ (ap\ (c\_2Ecanonical\_2ESPconst\ A\_27a)\ V2c)) = V2c))) \wedge ((\forall V3vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V4i \in ty\_2Equote\_2Eindex.((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_sp}\ A\_27a)\ V0r)\ V3vm)\ (ap\ (c\_2Ecanonical\_2ESPvar\ A\_27a)\ V4i)) = (ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a)\ V4i)\ V3vm)))) \wedge ((\forall V5vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V6p1 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V7p2 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_sp}\ A\_27a)\ V0r)\ V5vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPplus\ A\_27a)\ V6p1)\ V7p2)) = (ap\ (ap\ (ap\ (c\_2Ering\_2Ering\_\_RP\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_sp}\ A\_27a)\ V0r)\ V5vm)\ V6p1))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_sp}\ A\_27a)\ V0r)\ V5vm)\ V7p2)))))) \wedge (\forall V8vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V9p1 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V10p2 \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_sp}\ A\_27a)\ V0r)\ V8vm)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPmult\ A\_27a)\ V9p1)\ V10p2)) = (ap\ (ap\ (ap\ (c\_2Ering\_2Ering\_\_RM\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_sp}\ A\_27a)\ V0r)\ V8vm)\ V9p1))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EringNorm\_2Er\_\_interp\_\_sp}\ A\_27a)\ V0r)\ V8vm)\ V10p2))))))))))$$