

thm_2EringNorm_2Einterp_vl_def (TMSQzErN-
NDcjRSDNCjYhhXzJQjUPW56phBV)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (2)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (3)$$

Let $ty_2Ering_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ering_2Ering\ A0) \quad (4)$$

Let $c_2Ering_2Ering_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RM\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Ering_2Ering\ A_27a)}) \quad (5)$$

Let $c_2Ering_2Ering_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RP\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Ering_2Ering\ A_27a)}) \quad (6)$$

Let $c_2Ering_2Ering_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R1\ A_27a \in (A_27a^{(ty_2Ering_2Ering\ A_27a)}) \quad (7)$$

Let $c_2Ering_2Ering_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R0\ A_27a \in (A_27a^{(ty_2Ering_2Ering\ A_27a)}) \quad (8)$$

Let $ty_2Esemi_ring_2Esemi_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Esemi_ring_2Esemi_ring\ A0) \quad (9)$$

Let $ty_2Equote_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty_2Equote_2Eindex \quad (10)$$

Let $ty_2Equote_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Equote_2Evarmap\ A0) \quad (11)$$

Let $c_2Ecanonical_2Einterp_vl : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ecanonical_2Einterp_vl\ A_27a \in \\ (((A_27a^{(ty_2Elist_2Elist\ ty_2Equote_2Eindex)})^{(ty_2Equote_2Evarmap\ A_27a)})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (12)$$

Let $c_2Esemi_ring_2Erecordtype_2Esemi_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a \in \\ ((((((ty_2Esemi_ring_2Esemi_ring\ A_27a)^{(A_27a^{A_27a})})^{(A_27a^{A_27a})})^{(A_27a^{A_27a})})^{(A_27a^{A_27a})})^{(A_27a^{A_27a})}) \quad (13)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2E2 to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x)))$

Definition 4 We define $c_2Ering_2Esemi_ring_of$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ering_2Ering\ A_27a).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x)))$

Definition 5 We define $c_2EringNorm_2Er_interp_vl$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ering_2Ering\ A_27a).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x)))$

Let $c_2Ecanonical_2Eivl_aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ecanonical_2Eivl_aux\ A_27a \in \\ (((((A_27a^{(ty_2Elist_2Elist\ ty_2Equote_2Eindex)})^{(ty_2Equote_2Eindex)})^{(ty_2Equote_2Evarmap\ A_27a)})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (14)$$

Definition 6 We define $c_2EringNorm_2Er_ivl_aux$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ering_2Ering\ A_27a).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x)))$

Let $c_2Esemi_ring_2Esemi_ring_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRM\ A_27a \in \\ (((A_27a^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (15)$$

Let $c_2Esemi_ring_2Esemi_ring_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRP \\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \end{aligned} \quad (16)$$

Let $c_2Esemi_ring_2Esemi_ring_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR1 \\ A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \end{aligned} \quad (17)$$

Let $c_2Esemi_ring_2Esemi_ring_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR0 \\ A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \end{aligned} \quad (18)$$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0sr \in (ty_2Esemi_ring_2Esemi_ring \\ A_27a).(\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a).((ap (ap (\\ ap (c_2Ecanonical_2Einterp_vl\ A_27a)\ V0sr)\ V1vm)\ (c_2Elist_2ENIL \\ ty_2Equote_2Eindex)) = (ap (c_2Esemi_ring_2Esemi_ring_SR1 \\ A_27a)\ V0sr)))) \wedge (\forall V2sr \in (ty_2Esemi_ring_2Esemi_ring \\ A_27a).(\forall V3vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V4x \in \\ ty_2Equote_2Eindex.(\forall V5t \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex). \\ ((ap (ap (ap (c_2Ecanonical_2Einterp_vl\ A_27a)\ V2sr)\ V3vm)\ (ap \\ (ap (c_2Elist_2ECONS\ ty_2Equote_2Eindex)\ V4x)\ V5t)) = (ap (ap (\\ ap (ap (c_2Ecanonical_2Einv_aux\ A_27a)\ V2sr)\ V3vm)\ V4x)\ V5t))))))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in A_27a. (\forall V1a0 \in \\
& A_27a. (\forall V2f \in ((A_27a^{A_27a})^{A_27a}). (\forall V3f0 \in ((A_27a^{A_27a})^{A_27a}). \\
& ((ap\ (c_2Esemi_ring_2Esemi_ring_SR0\ A_27a)\ (ap\ (ap\ (ap\ (ap \\
& (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V0a)\ V1a0) \\
& V2f)\ V3f0)) = V0a)))) \wedge ((\forall V4a \in A_27a. (\forall V5a0 \in A_27a. \\
& (\forall V6f \in ((A_27a^{A_27a})^{A_27a}). (\forall V7f0 \in ((A_27a^{A_27a})^{A_27a}). \\
& ((ap\ (c_2Esemi_ring_2Esemi_ring_SR1\ A_27a)\ (ap\ (ap\ (ap\ (ap \\
& (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V4a)\ V5a0) \\
& V6f)\ V7f0)) = V5a0)))) \wedge ((\forall V8a \in A_27a. (\forall V9a0 \in A_27a. \\
& (\forall V10f \in ((A_27a^{A_27a})^{A_27a}). (\forall V11f0 \in ((A_27a^{A_27a})^{A_27a}). \\
& ((ap\ (c_2Esemi_ring_2Esemi_ring_SRP\ A_27a)\ (ap\ (ap\ (ap\ (ap \\
& (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V8a)\ V9a0) \\
& V10f)\ V11f0)) = V10f)))) \wedge ((\forall V12a \in A_27a. (\forall V13a0 \in \\
& A_27a. (\forall V14f \in ((A_27a^{A_27a})^{A_27a}). (\forall V15f0 \in ((\\
& A_27a^{A_27a})^{A_27a}). ((ap\ (c_2Esemi_ring_2Esemi_ring_SRM \\
& A_27a)\ (ap\ (ap\ (ap\ (ap\ (c_2Esemi_ring_2Erecordtype_2Esemi_ring \\
& A_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0)))))))))
\end{aligned} \tag{21}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (ty_2Ering_2Ering \\
& A_27a). ((\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a). ((ap\ (ap \\
& (ap\ (c_2EringNorm_2Er_interp_vl\ A_27a)\ V0r)\ V1vm)\ (c_2Elist_2ENIL \\
& ty_2Equote_2Eindex)) = (ap\ (c_2Ering_2Ering_R1\ A_27a)\ V0r))) \wedge \\
& (\forall V2vm \in (ty_2Equote_2Evarmap\ A_27a). (\forall V3x \in ty_2Equote_2Eindex. \\
& (\forall V4t \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex). ((ap\ (ap \\
& (ap\ (c_2EringNorm_2Er_interp_vl\ A_27a)\ V0r)\ V2vm)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& ty_2Equote_2Eindex)\ V3x)\ V4t)) = (ap\ (ap\ (ap\ (ap\ (c_2EringNorm_2Er_ivl_aux \\
& A_27a)\ V0r)\ V2vm)\ V3x)\ V4t)))))))))
\end{aligned}$$