

thm_2EringNorm_2Eivl__aux__def
(TMWo3r3wEommPy2GRwyWbU7SpkP2T8JBSSn)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (2)$$

Let $ty_2Equote_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Equote_2Evarmap\ A0) \quad (3)$$

Let $ty_2Equote_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty_2Equote_2Eindex \quad (4)$$

Let $c_2Equote_2Evarmap_find : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Equote_2Evarmap_find\ A.27a \in ((A.27a)^{(ty_2Equote_2Evarmap\ A.27a)})^{ty_2Equote_2Eindex} \quad (5)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (6)$$

Let $ty_2Ering_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ering_2Ering\ A0) \quad (7)$$

Let $c_2Ering_2Ering_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ering_2Ering_RM\ A.27a \in (((A.27a)^{A.27a})^{A.27a})^{(ty_2Ering_2Ering\ A.27a)} \quad (8)$$

Let $c_2Ering_2Ering_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RP\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Ering_2Ering\ A_27a)}) \quad (9)$$

Let $c_2Ering_2Ering_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R1\ A_27a \in (A_27a^{(ty_2Ering_2Ering\ A_27a)}) \quad (10)$$

Let $c_2Ering_2Ering_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R0\ A_27a \in (A_27a^{(ty_2Ering_2Ering\ A_27a)}) \quad (11)$$

Let $ty_2Esemi_ring_2Esemi_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Esemi_ring_2Esemi_ring\ A0) \quad (12)$$

Let $c_2Esemi_ring_2Erecordtype_2Esemi_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a \in (((((ty_2Esemi_ring_2Esemi_ring\ A_27a)^{(A_27a^{A_27a})^{A_27a}})^{(A_27a^{A_27a})^{A_27a}})^{A_27a})^{A_27a}) \quad (13)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x)))$

Definition 4 We define $c_2Ering_2Esemi_ring_of$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ering_2Ering\ A_27a).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x)))$

Let $c_2Ecanonical_2Eivl_aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ecanonical_2Eivl_aux\ A_27a \in (((A_27a^{(ty_2Elist_2Elist\ ty_2Equote_2Eindex)}^{ty_2Equote_2Eindex})^{(ty_2Equote_2Evarmap\ A_27a)})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (14)$$

Definition 5 We define $c_2EringNorm_2Er_ivl_aux$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ering_2Ering\ A_27a).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x)))$

Let $c_2Esemi_ring_2Esemi_ring_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRM\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (15)$$

Let $c_2Esemi_ring_2Esemi_ring_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRP\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (16)$$

Let $c_2Esemi_ring_2Esemi_ring_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR1 \\ A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \end{aligned} \quad (17)$$

Let $c_2Esemi_ring_2Esemi_ring_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR0 \\ A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \end{aligned} \quad (18)$$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0sr \in (ty_2Esemi_ring_2Esemi_ring \\ A_27a).(\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V2x \in \\ ty_2Equote_2Eindex.((ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl_aux \\ A_27a)\ V0sr)\ V1vm)\ V2x)\ (c_2Elist_2ENIL\ ty_2Equote_2Eindex)) = \\ (ap\ (ap\ (c_2Equote_2Evarmap_find\ A_27a)\ V2x)\ V1vm)))))) \wedge (\forall V3sr \in \\ (ty_2Esemi_ring_2Esemi_ring\ A_27a).(\forall V4vm \in (ty_2Equote_2Evarmap \\ A_27a).(\forall V5x \in ty_2Equote_2Eindex.(\forall V6x_27 \in ty_2Equote_2Eindex. \\ (\forall V7t_27 \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex).((ap \\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl_aux\ A_27a)\ V3sr)\ V4vm)\ V5x)\ (\\ ap\ (ap\ (c_2Elist_2ECONS\ ty_2Equote_2Eindex)\ V6x_27)\ V7t_27)) = \\ (ap\ (ap\ (ap\ (c_2Esemi_ring_2Esemi_ring_SRM\ A_27a)\ V3sr)\ (ap \\ (ap\ (c_2Equote_2Evarmap_find\ A_27a)\ V5x)\ V4vm))\ (ap\ (ap\ (ap\ (ap \\ (c_2Ecanonical_2Eivl_aux\ A_27a)\ V3sr)\ V4vm)\ V6x_27)\ V7t_27)))))))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in A_27a. (\forall V1a0 \in \\
& A_27a. (\forall V2f \in ((A_27a^{A_27a})^{A_27a}). (\forall V3f0 \in ((A_27a^{A_27a})^{A_27a}). \\
& ((ap\ (c_2Esemi_ring_2Esemi_ring_SR0\ A_27a)\ (ap\ (ap\ (ap\ (ap \\
& (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V0a)\ V1a0) \\
& V2f)\ V3f0)) = V0a)))) \wedge ((\forall V4a \in A_27a. (\forall V5a0 \in A_27a. \\
& (\forall V6f \in ((A_27a^{A_27a})^{A_27a}). (\forall V7f0 \in ((A_27a^{A_27a})^{A_27a}). \\
& ((ap\ (c_2Esemi_ring_2Esemi_ring_SR1\ A_27a)\ (ap\ (ap\ (ap\ (ap \\
& (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V4a)\ V5a0) \\
& V6f)\ V7f0)) = V5a0)))) \wedge ((\forall V8a \in A_27a. (\forall V9a0 \in A_27a. \\
& (\forall V10f \in ((A_27a^{A_27a})^{A_27a}). (\forall V11f0 \in ((A_27a^{A_27a})^{A_27a}). \\
& ((ap\ (c_2Esemi_ring_2Esemi_ring_SRP\ A_27a)\ (ap\ (ap\ (ap\ (ap \\
& (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V8a)\ V9a0) \\
& V10f)\ V11f0)) = V10f)))) \wedge ((\forall V12a \in A_27a. (\forall V13a0 \in \\
& A_27a. (\forall V14f \in ((A_27a^{A_27a})^{A_27a}). (\forall V15f0 \in ((\\
& A_27a^{A_27a})^{A_27a}). ((ap\ (c_2Esemi_ring_2Esemi_ring_SRM \\
& A_27a)\ (ap\ (ap\ (ap\ (ap\ (c_2Esemi_ring_2Erecordtype_2Esemi_ring \\
& A_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0)))))))))
\end{aligned} \tag{21}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (ty_2Ering_2Ering \\
& A_27a). ((\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a). (\forall V2x \in \\
& ty_2Equote_2Eindex. ((ap\ (ap\ (ap\ (ap\ (c_2EringNorm_2Er_ivl_aux \\
& A_27a)\ V0r)\ V1vm)\ V2x)\ (c_2Elist_2ENIL\ ty_2Equote_2Eindex)) = \\
& (ap\ (ap\ (c_2Equote_2Evarmap_find\ A_27a)\ V2x)\ V1vm)))) \wedge ((\forall V3vm \in \\
& (ty_2Equote_2Evarmap\ A_27a). (\forall V4x \in ty_2Equote_2Eindex. \\
& (\forall V5x_27 \in ty_2Equote_2Eindex. (\forall V6t_27 \in (ty_2Elist_2Elist \\
& ty_2Equote_2Eindex). ((ap\ (ap\ (ap\ (ap\ (c_2EringNorm_2Er_ivl_aux \\
& A_27a)\ V0r)\ V3vm)\ V4x)\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Equote_2Eindex) \\
& V5x_27)\ V6t_27)) = (ap\ (ap\ (ap\ (c_2Ering_2Ering_RM\ A_27a)\ V0r) \\
& (ap\ (ap\ (c_2Equote_2Evarmap_find\ A_27a)\ V4x)\ V3vm))\ (ap\ (ap\ (ap \\
& (ap\ (c_2EringNorm_2Er_ivl_aux\ A_27a)\ V0r)\ V3vm)\ V5x_27)\ V6t_27)))))))))
\end{aligned}$$