

thm\_2EringNorm\_2Emonom\_insert\_def  
(TMYksGfYnZKfJDwccFBhhycSYr-  
WCwyo8QF)

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Let  $ty\_2Ecanonical\_2Ecanonical\_sum : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_sum\ A0) \quad (1)$$

Let  $c\_2Ecanonical\_2ENil\_monom : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_sum\ A\_27a) \quad (2)$$

Let  $ty\_2Equote\_2Eindex : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \quad (3)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (4)$$

Let  $c\_2Ecanonical\_2Econs\_varlist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Econs\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (5)$$

Let  $ty\_2EternaryComparisons\_2Eordering : \iota$  be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \quad (6)$$

Let  $c\_2Equote\_2Eindex\_compare : \iota$  be given. Assume the following.

$$c\_2Equote\_2Eindex\_compare \in ((ty\_2EternaryComparisons\_2Eordering)^{ty\_2Equote\_2Eindex})^{ty\_2Equote\_2Eindex} \quad (7)$$

Let  $c\_2EternaryComparisons\_2Elist\_compare : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EternaryComparisons\_2Elist\_compare\ A\_27a\ A\_27b \in (((ty\_2EternaryComparisons\_2Eordering^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27b)} \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2EternaryComparisons\_2Eordering2num : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2Eordering2num \in (ty\_2Enum\_2Enum^{ty\_2EternaryComparisons\_2Eordering}) \quad (10)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (11)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (12)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (14)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A\_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ c\_2Enum\_2EREP\_num\ (ap\ c\_2Enum\_2ESUC\_REP)))$

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E2B\ (ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Enum\_2EREP\_num\ (ap\ c\_2Enum\_2ESUC\_REP))))$

**Definition 8** We define  $c\_Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E\_21\ 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21\ 2) (\lambda V2t \in 2.V2t))))$

**Definition 12** We define  $c\_Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge P x)$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_Emin\_2E\_40) (ap P x))))))$

**Definition 14** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2E\_21\ 2))$

**Definition 15** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_Emin\_2E\_40) (ap P x))))$

**Definition 16** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0m$

**Definition 17** We define  $c\_EternaryComparisons\_2Eordering\_CASE$  to be  $\lambda A\_27a : \iota.\lambda V0x \in ty\_2EternaryComparisons\_2Eordering\_CASE$

Let  $c\_2Ecanonical\_2ECons\_monom : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_monom\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_sum\ A\_27a)})(ty\_2Elist\_2Elist\ ty\_2Eq\ A\_27a)) \quad (16)$$

Let  $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \quad (17)$$

Let  $c\_2Ering\_2Ering\_RM : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (18)$$

Let  $c\_2Ering\_2Ering\_RP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (19)$$

Let  $c\_2Ering\_2Ering\_R1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (20)$$

Let  $c\_2Ering\_2Ering\_R0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (21)$$

Let  $ty\_2Esemi\_ring\_2Esemi\_ring : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_ring\_2Esemi\_ring\ A0) \quad (22)$$

Let  $c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring\ A\_27a \in (((((ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)^{(A\_27a^{A\_27a}})^{A\_27a})^{(A\_27a^{A\_27a}})^{A\_27a})^{A\_27a})^{A\_27a})^{A\_27a}) \quad (23)$$

**Definition 18** We define  $c\_2Ering\_2Esemi\_ring\_of$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Let  $c\_2Ecanonical\_2Emonom\_insert : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Emonom\_insert\ A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_sum\ A\_27a)}(ty\_2Elist\_2Elist\ A\_27a))^{A\_27a})^{A\_27a})^{A\_27a})^{A\_27a}) \quad (24)$$

**Definition 19** We define  $c\_2EringNorm\_2Er\_monom\_insert$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRM : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)}) \quad (25)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)}) \quad (26)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)}) \quad (27)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)}) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0t2 \in (ty\_2Ecanonical\_2Ecanonical\_sum \\
& \quad A.27a).(\forall V1sr \in (ty\_2Esemi\_ring\_2Esemi\_ring A.27a). \\
& \quad (\forall V2l2 \in (ty\_2Elist\_2Elist ty\_2Equote\_2Eindex).(\forall V3l1 \in \\
& \quad \quad (ty\_2Elist\_2Elist ty\_2Equote\_2Eindex).(\forall V4c2 \in A.27a. \\
& \quad \quad (\forall V5c1 \in A.27a.((ap (ap (ap (ap (c\_2Ecanonical\_2Emonom\_insert \\
& \quad \quad A.27a) V1sr) V5c1) V3l1) (ap (ap (ap (c\_2Ecanonical\_2ECons\_monom \\
& \quad \quad A.27a) V4c2) V2l2) V0t2)) = (ap (ap (ap (ap (c\_2EternaryComparisons\_2Eordering\_CASE \\
& \quad (ty\_2Ecanonical\_2Ecanonical\_sum A.27a)) (ap (ap (ap (c\_2EternaryComparisons\_2Elist\_compare \\
& \quad ty\_2Equote\_2Eindex ty\_2Equote\_2Eindex) c\_2Equote\_2Eindex\_compare) \\
& \quad V3l1) V2l2)) (ap (ap (ap (c\_2Ecanonical\_2ECons\_monom A.27a) V5c1) \\
& \quad V3l1) (ap (ap (ap (c\_2Ecanonical\_2ECons\_monom A.27a) V4c2) V2l2) \\
& \quad V0t2))) (ap (ap (ap (c\_2Ecanonical\_2ECons\_monom A.27a) (ap (ap \\
& \quad (ap (c\_2Esemi\_ring\_2Esemi\_ring\_SRP A.27a) V1sr) V5c1) V4c2)) \\
& \quad V3l1) V0t2)) (ap (ap (ap (c\_2Ecanonical\_2ECons\_monom A.27a) V4c2) \\
& \quad V2l2) (ap (ap (ap (ap (c\_2Ecanonical\_2Emonom\_insert A.27a) V1sr) \\
& \quad V5c1) V3l1) V0t2))))))))) \wedge ((\forall V6t2 \in (ty\_2Ecanonical\_2Ecanonical\_sum \\
& \quad A.27a).(\forall V7sr \in (ty\_2Esemi\_ring\_2Esemi\_ring A.27a). \\
& \quad (\forall V8l2 \in (ty\_2Elist\_2Elist ty\_2Equote\_2Eindex).(\forall V9l1 \in \\
& \quad \quad (ty\_2Elist\_2Elist ty\_2Equote\_2Eindex).(\forall V10c1 \in A.27a. \\
& \quad \quad ((ap (ap (ap (ap (c\_2Ecanonical\_2Emonom\_insert A.27a) V7sr) V10c1) \\
& \quad \quad V9l1) (ap (ap (c\_2Ecanonical\_2ECons\_varlist A.27a) V8l2) V6t2)) = \\
& \quad (ap (ap (ap (ap (c\_2EternaryComparisons\_2Eordering\_CASE (ty\_2Ecanonical\_2Ecanonical\_sum \\
& \quad \quad A.27a)) (ap (ap (ap (c\_2EternaryComparisons\_2Elist\_compare \\
& \quad ty\_2Equote\_2Eindex ty\_2Equote\_2Eindex) c\_2Equote\_2Eindex\_compare) \\
& \quad V9l1) V8l2)) (ap (ap (ap (c\_2Ecanonical\_2ECons\_monom A.27a) V10c1) \\
& \quad V9l1) (ap (ap (c\_2Ecanonical\_2ECons\_varlist A.27a) V8l2) V6t2))) \\
& \quad (ap (ap (ap (c\_2Ecanonical\_2ECons\_monom A.27a) (ap (ap (ap (c\_2Esemi\_ring\_2Esemi\_ring\_SRP \\
& \quad \quad A.27a) V7sr) V10c1) (ap (c\_2Esemi\_ring\_2Esemi\_ring\_SR1 A.27a) \\
& \quad \quad V7sr))) V9l1) V6t2)) (ap (ap (c\_2Ecanonical\_2ECons\_varlist A.27a) \\
& \quad V8l2) (ap (ap (ap (ap (c\_2Ecanonical\_2Emonom\_insert A.27a) V7sr) \\
& \quad V10c1) V9l1) V6t2))))))))) \wedge ((\forall V11sr \in (ty\_2Esemi\_ring\_2Esemi\_ring \\
& \quad A.27a).(\forall V12l1 \in (ty\_2Elist\_2Elist ty\_2Equote\_2Eindex). \\
& \quad (\forall V13c1 \in A.27a.((ap (ap (ap (ap (c\_2Ecanonical\_2Emonom\_insert \\
& \quad \quad A.27a) V11sr) V13c1) V12l1) (c\_2Ecanonical\_2ENil\_monom A.27a)) = \\
& \quad (ap (ap (ap (c\_2Ecanonical\_2ECons\_monom A.27a) V13c1) V12l1) \\
& \quad \quad (c\_2Ecanonical\_2ENil\_monom A.27a)))))))))
\end{aligned}$$

(30)

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0a \in A\_27a. (\forall V1a0 \in \\
& A\_27a. (\forall V2f \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V3f0 \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& ((\text{ap } (c\_2Esemi\_ring\_2Esemi\_ring\_SR0 \ A\_27a) \ (\text{ap } (\text{ap } (\text{ap } (\text{ap} \\
& (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \ A\_27a) \ V0a) \ V1a0) \\
& \ V2f) \ V3f0)) = V0a)))) \wedge ((\forall V4a \in A\_27a. (\forall V5a0 \in A\_27a. \\
& (\forall V6f \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V7f0 \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& ((\text{ap } (c\_2Esemi\_ring\_2Esemi\_ring\_SR1 \ A\_27a) \ (\text{ap } (\text{ap } (\text{ap } (\text{ap} \\
& (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \ A\_27a) \ V4a) \ V5a0) \\
& \ V6f) \ V7f0)) = V5a0)))) \wedge ((\forall V8a \in A\_27a. (\forall V9a0 \in A\_27a. \\
& (\forall V10f \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V11f0 \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& ((\text{ap } (c\_2Esemi\_ring\_2Esemi\_ring\_SRP \ A\_27a) \ (\text{ap } (\text{ap } (\text{ap } (\text{ap} \\
& (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \ A\_27a) \ V8a) \ V9a0) \\
& \ V10f) \ V11f0)) = V10f)))) \wedge (\forall V12a \in A\_27a. (\forall V13a0 \in \\
& \ A\_27a. (\forall V14f \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V15f0 \in (( \\
& \ A\_27a^{A\_27a})^{A\_27a}). ((\text{ap } (c\_2Esemi\_ring\_2Esemi\_ring\_SRM \\
& \ A\_27a) \ (\text{ap } (\text{ap } (\text{ap } (\text{ap } (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \ A\_27a) \ V12a) \ V13a0) \ V14f) \ V15f0)) = V15f0)))))))))
\end{aligned} \tag{31}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering\ A.27a).((\forall V1t2 \in (ty\_2Ecanonical\_2Ecanonical\_sum\ A.27a). \\
& (\forall V2l2 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V3l1 \in \\
& (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V4c2 \in A.27a. \\
& (\forall V5c1 \in A.27a.((ap\ (ap\ (ap\ (ap\ (c\_2EringNorm\_2Er\_monom\_insert \\
& A.27a)\ V0r)\ V5c1)\ V3l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_monom \\
& A.27a)\ V4c2)\ V2l2)\ V1t2)) = (ap\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eordering\_CASE \\
& (ty\_2Ecanonical\_2Ecanonical\_sum\ A.27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_compare \\
& ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_compare) \\
& V3l1)\ V2l2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_monom\ A.27a)\ V5c1) \\
& V3l1)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_monom\ A.27a)\ V4c2)\ V2l2) \\
& V1t2)))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_monom\ A.27a)\ (ap\ (ap \\
& (ap\ (c\_2Ering\_2Ering\_RP\ A.27a)\ V0r)\ V5c1)\ V4c2))\ V3l1)\ V1t2))) \\
& (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_monom\ A.27a)\ V4c2)\ V2l2)\ (ap \\
& (ap\ (ap\ (ap\ (c\_2EringNorm\_2Er\_monom\_insert\ A.27a)\ V0r)\ V5c1) \\
& V3l1)\ V1t2))))))\ (\forall V6t2 \in (ty\_2Ecanonical\_2Ecanonical\_sum \\
& A.27a).(\forall V7l2 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex). \\
& (\forall V8l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V9c1 \in \\
& A.27a.((ap\ (ap\ (ap\ (ap\ (c\_2EringNorm\_2Er\_monom\_insert\ A.27a) \\
& V0r)\ V9c1)\ V8l1)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_varlist\ A.27a) \\
& V7l2)\ V6t2)) = (ap\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eordering\_CASE \\
& (ty\_2Ecanonical\_2Ecanonical\_sum\ A.27a))\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_compare \\
& ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_compare) \\
& V8l1)\ V7l2))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_monom\ A.27a)\ V9c1) \\
& V8l1)\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_varlist\ A.27a)\ V7l2)\ V6t2))) \\
& (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_monom\ A.27a)\ (ap\ (ap\ (ap\ (c\_2Ering\_2Ering\_RP \\
& A.27a)\ V0r)\ V9c1)\ (ap\ (c\_2Ering\_2Ering\_R1\ A.27a)\ V0r)))\ V8l1) \\
& V6t2))\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_varlist\ A.27a)\ V7l2)\ (ap \\
& (ap\ (ap\ (ap\ (c\_2EringNorm\_2Er\_monom\_insert\ A.27a)\ V0r)\ V9c1) \\
& V8l1)\ V6t2))))))\ (\forall V10l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex). \\
& (\forall V11c1 \in A.27a.((ap\ (ap\ (ap\ (ap\ (c\_2EringNorm\_2Er\_monom\_insert \\
& A.27a)\ V0r)\ V11c1)\ V10l1)\ (c\_2Ecanonical\_2ENil\_monom\ A.27a)) = \\
& (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_monom\ A.27a)\ V11c1)\ V10l1) \\
& (c\_2Ecanonical\_2ENil\_monom\ A.27a))))))
\end{aligned}$$