thm_2EringNorm_2Espolynom__normalize__def (TMdue4vydaqQ88TBCxY6jhb8wKgvwDAg8fE)

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Let $ty\_2Ecanonical\_2Espolynom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Espolynom\ A0) \quad (1)$$

Let $c\_2Ecanonical\_2ESPmult : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPmult\ A\_27a \in (((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)}) \quad (2)$$

Let $c\_2Ecanonical\_2ESPplus : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPplus\ A\_27a \in (((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)}) \quad (3)$$

Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \quad (4)$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \quad (5)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (6)$$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \quad (7)$$

Let $c\_2Ecanonical\_2ESPconst : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPconst\ A\_27a \in ((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{A\_27a}) \quad (8)$$

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \quad (9)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (10)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (11)$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli} \quad (12)$$

Let $c\_2Ecanonical\_2ESPvar : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ESPvar\ A\_27a \in ((ty\_2Ecanonical\_2Espolynom\ A\_27a)^{ty\_2Equote\_2Eindex}) \quad (13)$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \quad (14)$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Let $c\_2Ecanonical\_2Espolynom\_\_normalize : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Espolynom\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2E} \quad (15)$$

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \quad (16)$$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{17}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{18}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{19}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{20}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{21}$$

Definition 6 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap$

Definition 7 *We define* c_2EringNorm_2Er__spolynom__normalize *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Eri$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \tag{22}$$

Definition 8 *We define* c_2EringNorm_2Er__canonical__sum__prod *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Eri$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \tag{23}$$

Definition 9 *We define* c_2EringNorm_2Er__canonical__sum__merge *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2E$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1i \in ty\_2Equote\_2Eindex.((ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V0sr)\ (ap\ (c\_2Ecanonical\_2ESPvar\ A\_27a)\ V1i)) = (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Equote\_2Eindex)\ V1i)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex)))\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))\wedge((\forall V2sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V3c \in A\_27a.((ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V2sr)\ (ap\ (c\_2Ecanonical\_2ESPconst\ A\_27a)\ V3c)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ A\_27a)\ V3c)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex))\ (c\_2Ecanonical\_2ENil\_\_monom\ A\_27a)))))\wedge ((\forall V4sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V5l \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V6r \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V4sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPplus\ A\_27a)\ V5l)\ V6r)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge\ A\_27a)\ V4sr)\ (ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V4sr)\ V5l))\ (ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V4sr)\ V6r))))))\wedge (\forall V7sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V8l \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).(\forall V9r \in (ty\_2Ecanonical\_2Espolynom\ A\_27a).((ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V7sr)\ (ap\ (ap\ (c\_2Ecanonical\_2ESPmult\ A\_27a)\ V8l)\ V9r)) = (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod\ A\_27a)\ V7sr)\ (ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V7sr)\ V8l))\ (ap\ (ap\ (c\_2Ecanonical\_2Espolynom\_\_normalize\ A\_27a)\ V7sr)\ V9r))))))))) \tag{25}$$

Theorem 1

∀A_27a.nonempty A_27a⇒(∀V0r ∈ (ty_2Ering_2Ering
A_27a).((∀V1i ∈ ty_2Equote_2Eindex.((ap (ap (c_2EringNorm_2Er__spolynom__normalize
A_27a) V0r) (ap (c_2Ecanonical_2ESPvar A_27a) V1i)) = (ap (ap (c_2Ecanonical_2ECons__varlist
A_27a) (ap (ap (c_2Elist_2ECONS ty_2Equote_2Eindex) V1i) (c_2Elist_2ENIL
ty_2Equote_2Eindex))) (c_2Ecanonical_2ENil__monom A_27a))))∧
((∀V2c ∈ A_27a.((ap (ap (c_2EringNorm_2Er__spolynom__normalize
A_27a) V0r) (ap (c_2Ecanonical_2ESPconst A_27a) V2c)) = (ap (ap
(ap (c_2Ecanonical_2ECons__monom A_27a) V2c) (c_2Elist_2ENIL
ty_2Equote_2Eindex)) (c_2Ecanonical_2ENil__monom A_27a))))∧
((∀V3l ∈ (ty_2Ecanonical_2Espolynom A_27a).(∀V4r_27 ∈
(ty_2Ecanonical_2Espolynom A_27a).((ap (ap (c_2EringNorm_2Er__spolynom__normalize
A_27a) V0r) (ap (ap (c_2Ecanonical_2ESPplus A_27a) V3l) V4r_27)) =
(ap (ap (ap (c_2EringNorm_2Er__canonical__sum__merge A_27a)
V0r) (ap (ap (c_2EringNorm_2Er__spolynom__normalize A_27a) V0r)
V3l)) (ap (ap (c_2EringNorm_2Er__spolynom__normalize A_27a)
V0r) V4r_27)))))∧(∀V5l ∈ (ty_2Ecanonical_2Espolynom A_27a).
(∀V6r_27 ∈ (ty_2Ecanonical_2Espolynom A_27a).((ap (ap
(c_2EringNorm_2Er__spolynom__normalize A_27a) V0r) (ap (ap (
c_2Ecanonical_2ESPmult A_27a) V5l) V6r_27)) = (ap (ap (ap (c_2EringNorm_2Er__canonical__sum__proc
A_27a) V0r) (ap (ap (c_2EringNorm_2Er__spolynom__normalize A_27a)
V0r) V5l)) (ap (ap (c_2EringNorm_2Er__spolynom__normalize A_27a)
V0r) V6r_27)))))))))