

thm_2Ering_2Emult__assoc (TM-
SiE648Mo75nvJxyuphDCrAPZSEWoXH6ZM)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Let $ty_2Ering_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ering_2Ering A0) \quad (1)$$

Let $c_2Ering_2Ering_RN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ering_2Ering_RN A_27a \in ((A_27a^{A_27a})^{(ty_2Ering_2Ering A_27a)}) \quad (2)$$

Let $c_2Ering_2Ering_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ering_2Ering_R1 A_27a \in (A_27a^{(ty_2Ering_2Ering A_27a)}) \quad (3)$$

Let $c_2Ering_2Ering_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ering_2Ering_R0 A_27a \in (A_27a^{(ty_2Ering_2Ering A_27a)}) \quad (4)$$

Let $c_2Ering_2Ering_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ering_2Ering_RM A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Ering_2Ering A_27a)}) \quad (5)$$

Let $c_2Ering_2Ering_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ering_2Ering_RP A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Ering_2Ering A_27a)}) \quad (6)$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 6 We define $c_2Ering_2Eis_ring$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ering_2Ering A_27a).(ap (ap c_2$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0r \in (ty_2Ering_2Ering \\ & A_27a).(p (ap (c_2Ering_2Eis_ring A_27a) V0r)) \Rightarrow (\forall V1n \in \\ & A_27a.(\forall V2m \in A_27a.(\forall V3p \in A_27a.((ap (ap (ap (c_2Ering_2Ering_RM \\ & A_27a) V0r) V1n) (ap (ap (ap (c_2Ering_2Ering_RM A_27a) V0r) V2m) \\ & V3p)) = (ap (ap (ap (c_2Ering_2Ering_RM A_27a) V0r) (ap (ap (ap (\\ & c_2Ering_2Ering_RM A_27a) V0r) V1n) V2m)) V3p)))))) \end{aligned}$$