

# thm\_2Ering\_2Emult\_one\_left (TMcMacHsYT- fRKpjhmqwbc6A7kFyE1pDt4vZ)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

Let  $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ering\_2Ering A0) \quad (1)$$

Let  $c\_2Ering\_2Ering\_RN : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_RN A\_27a \in ((A\_27a^{A\_27a})^{(ty\_2Ering\_2Ering A\_27a)}) \quad (2)$$

Let  $c\_2Ering\_2Ering\_R1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_R1 A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering A\_27a)}) \quad (3)$$

Let  $c\_2Ering\_2Ering\_R0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_R0 A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering A\_27a)}) \quad (4)$$

Let  $c\_2Ering\_2Ering\_RM : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_RM A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering A\_27a)}) \quad (5)$$

Let  $c\_2Ering\_2Ering\_RP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_RP A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering A\_27a)}) \quad (6)$$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 5** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in 2.$

**Definition 6** We define  $c\_2Ering\_2Eis\_ring$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering A\_27a).(ap (ap c\_2Ering\_2Eis\_ring A\_27a) V0r))$

**Theorem 1**

$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0r \in (ty\_2Ering\_2Ering A\_27a).(p (ap (c\_2Ering\_2Eis\_ring A\_27a) V0r)) \Rightarrow (\forall V1n \in A\_27a.((ap (ap (ap (c\_2Ering\_2Ering\_RM A\_27a) V0r) (ap (c\_2Ering\_2Ering\_R1 A\_27a) V0r)) V1n) = V1n))))$