

thm\_2Ering\_2Ering\_fn\_updates  
(TMR6YHGja2ATqxCW3uNV85THts5EocXN96c)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ering\_2Ering A0) \quad (1)$$

Let  $c\_2Ering\_2Ering\_R0\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_R0\_fupd A\_27a \in ((ty\_2Ering\_2Ering A\_27a)^{(ty\_2Ering\_2Ering A\_27a)})^{(A\_27a^{A\_27a})} \quad (2)$$

Let  $c\_2Ering\_2Ering\_R1\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_R1\_fupd A\_27a \in ((ty\_2Ering\_2Ering A\_27a)^{(ty\_2Ering\_2Ering A\_27a)})^{(A\_27a^{A\_27a})} \quad (3)$$

Let  $c\_2Ering\_2Ering\_RP\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_RP\_fupd A\_27a \in (((ty\_2Ering\_2Ering A\_27a)^{(ty\_2Ering\_2Ering A\_27a)})^{((A\_27a^{A\_27a})^{A\_27a})}^{((A\_27a^{A\_27a})^{A\_27a})}) \quad (4)$$

Let  $c\_2Ering\_2Ering\_RM\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ering\_2Ering\_RM\_fupd A\_27a \in (((ty\_2Ering\_2Ering A\_27a)^{(ty\_2Ering\_2Ering A\_27a)})^{((A\_27a^{A\_27a})^{A\_27a})}^{((A\_27a^{A\_27a})^{A\_27a})}) \quad (5)$$

Let  $c\_2Ering\_2Erecordtype\_2Ering : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Erecordtype\_2Ering\ A\_27a \in ((((((ty\_2Ering\_2Ering\ A\_27a)^{(A\_27a^{A\_27a}})})^{(A\_27a^{A\_27a})^{A\_27a}}))^{(A\_27a^{A\_27a})^{A\_27a}})_{A\_27a})_{A\_27a} \quad (6)$$

Let  $c\_2Ering\_2Ering\_RN\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_RN\_fupd\ A\_27a \in (((ty\_2Ering\_2Ering\ A\_27a)^{(ty\_2Ering\_2Ering\ A\_27a)})^{(A\_27a^{A\_27a})^{(A\_27a^{A\_27a})}}) \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f2 \in (A\_27a^{A\_27a}). (\forall V1a \in \\ & \quad A\_27a. (\forall V2a0 \in A\_27a. (\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\ & \quad (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V5f1 \in (A\_27a^{A\_27a}). \\ & \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_R0\_fupd\ A\_27a)\ V0f2)\ (ap\ (ap\ (ap\ (ap \\ & \quad (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0) \\ & \quad V5f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a) \\ & \quad (ap\ V0f2\ V1a))\ V2a0)\ V3f)\ V4f0)\ V5f1))))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f2 \in (A\_27a^{A\_27a}). (\forall V1a \in \\ & \quad A\_27a. (\forall V2a0 \in A\_27a. (\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\ & \quad (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V5f1 \in (A\_27a^{A\_27a}). \\ & \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_R1\_fupd\ A\_27a)\ V0f2)\ (ap\ (ap\ (ap\ (ap \\ & \quad (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0) \\ & \quad V5f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a) \\ & \quad V1a)\ (ap\ V0f2\ V2a0))\ V3f)\ V4f0)\ V5f1))))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f2 \in (((A\_27a^{A\_27a})_{A\_27a})^{(A\_27a^{A\_27a})^{A\_27a}}). \\ & \quad (\forall V1a \in A\_27a. (\forall V2a0 \in A\_27a. (\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\ & \quad (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V5f1 \in (A\_27a^{A\_27a}). \\ & \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_RP\_fupd\ A\_27a)\ V0f2)\ (ap\ (ap\ (ap\ (ap \\ & \quad (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0) \\ & \quad V5f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a) \\ & \quad V1a)\ V2a0)\ (ap\ V0f2\ V3f))\ V4f0)\ V5f1))))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f2 \in (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{A\_27a})^{A\_27a}})). \\
& \quad (\forall V1a \in A\_27a. (\forall V2a0 \in A\_27a. (\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad \quad (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V5f1 \in (A\_27a^{A\_27a}). \\
& \quad \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_RM\_fupd\ A\_27a)\ V0f2)\ (ap\ (ap\ (ap\ (ap \\
& \quad \quad (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0) \\
& \quad \quad V5f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a) \\
& \quad \quad V1a)\ V2a0)\ V3f)\ (ap\ V0f2\ V4f0))\ V5f1)))))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f2 \in ((A\_27a^{A\_27a})^{(A\_27a^{A\_27a})}). \\
& \quad (\forall V1a \in A\_27a. (\forall V2a0 \in A\_27a. (\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad \quad (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V5f1 \in (A\_27a^{A\_27a}). \\
& \quad \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_RN\_fupd\ A\_27a)\ V0f2)\ (ap\ (ap\ (ap\ (ap \\
& \quad \quad (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0) \\
& \quad \quad V5f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a) \\
& \quad \quad V1a)\ V2a0)\ V3f)\ V4f0)\ (ap\ V0f2\ V5f1)))))))))
\end{aligned} \tag{12}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f2 \in (A\_27a^{A\_27a}). ( \\
& \quad \forall V1a \in A\_27a. (\forall V2a0 \in A\_27a. (\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V5f1 \in (A\_27a^{A\_27a}). \\
& \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_R0\_fupd\ A\_27a)\ V0f2)\ (ap\ (ap\ (ap\ (ap \\
& \quad (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0) \\
& \quad V5f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a) \\
& \quad (ap\ V0f2\ V1a))\ V2a0)\ V3f)\ V4f0)\ V5f1)))))) \wedge ((\forall V6f2 \in (A\_27a^{A\_27a}). \\
& \quad (\forall V7a \in A\_27a. (\forall V8a0 \in A\_27a. (\forall V9f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad (\forall V10f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V11f1 \in (A\_27a^{A\_27a}). \\
& \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_R1\_fupd\ A\_27a)\ V6f2)\ (ap\ (ap\ (ap\ (ap \\
& \quad (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V7a)\ V8a0)\ V9f)\ V10f0) \\
& \quad V11f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a) \\
& \quad V7a)\ (ap\ V6f2\ V8a0))\ V9f)\ V10f0)\ V11f1)))))) \wedge ((\forall V12f2 \in \\
& \quad (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{A\_27a})^{A\_27a}}). (\forall V13a \in \\
& \quad A\_27a. (\forall V14a0 \in A\_27a. (\forall V15f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad (\forall V16f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V17f1 \in (A\_27a^{A\_27a}). \\
& \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_RP\_fupd\ A\_27a)\ V12f2)\ (ap\ (ap\ (ap\ ( \\
& \quad ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V13a)\ V14a0)\ V15f) \\
& \quad V16f0)\ V17f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering \\
& \quad A\_27a)\ V13a)\ V14a0)\ (ap\ V12f2\ V15f))\ V16f0)\ V17f1)))))) \wedge ((\forall V18f2 \in \\
& \quad (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{A\_27a})^{A\_27a}}). (\forall V19a \in \\
& \quad A\_27a. (\forall V20a0 \in A\_27a. (\forall V21f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad (\forall V22f0 \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V23f1 \in (A\_27a^{A\_27a}). \\
& \quad ((ap\ (ap\ (c\_2Ering\_2Ering\_RM\_fupd\ A\_27a)\ V18f2)\ (ap\ (ap\ (ap\ ( \\
& \quad ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V19a)\ V20a0)\ V21f) \\
& \quad V22f0)\ V23f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering \\
& \quad A\_27a)\ V19a)\ V20a0)\ V21f)\ (ap\ V18f2\ V22f0))\ V23f1)))))) \wedge ((\forall V24f2 \in \\
& \quad ((A\_27a^{A\_27a})^{(A\_27a^{A\_27a})}). (\forall V25a \in A\_27a. (\forall V26a0 \in \\
& \quad A\_27a. (\forall V27f \in ((A\_27a^{A\_27a})^{A\_27a}). (\forall V28f0 \in (( \\
& \quad A\_27a^{A\_27a})^{A\_27a}). (\forall V29f1 \in (A\_27a^{A\_27a}). ((ap\ (ap\ (c\_2Ering\_2Ering\_RN\_fupd \\
& \quad A\_27a)\ V24f2)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering \\
& \quad A\_27a)\ V25a)\ V26a0)\ V27f)\ V28f0)\ V29f1)) = (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering \\
& \quad A\_27a)\ V25a)\ V26a0)\ V27f)\ V28f0)\ (ap\ V24f2\ V29f1))))))))))
\end{aligned}$$