

thm_2Ering_2Ering_is_semi_ring (TMbBgTqf7P JrRduoWVorpgh2XBDcF3ca7G)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x)$

Definition 3 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Let `ty_2Ering_2Ering` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Ering_2Ering } A 0) \quad (1)$$

Let `c_2Ering_2Ering_RN` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Ering_2Ering_RN } A_27a \in ((A_27a^{A_27a})^{(\text{ty_2Ering_2Ering } A_27a)}) \quad (2)$$

Let `c_2Ering_2Ering_RM` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Ering_2Ering_RM } A_27a \in (((A_27a^{A_27a})^{A_27a})^{(\text{ty_2Ering_2Ering } A_27a)}) \quad (3)$$

Let `c_2Ering_2Ering_RP` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Ering_2Ering_RP } A_27a \in (((A_27a^{A_27a})^{A_27a})^{(\text{ty_2Ering_2Ering } A_27a)}) \quad (4)$$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V 0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Let `c_2Ering_2Ering_R0` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Ering_2Ering_R0 } A_27a \in (A_27a^{(\text{ty_2Ering_2Ering } A_27a)}) \quad (5)$$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V 2t \in 2. V 2t)))$

Let $c_2Ering_2Ering_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R1\ A_27a \in (A_27a^{(ty_2Ering_2Ering\ A_27a)}) \quad (6)$$

Definition 6 We define $c_2Ering_2Eis_ring$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ering_2Ering\ A_27a).(ap\ (ap\ c_2Ering_2Ering_R1\ V0r))$

Let $ty_2Esemi_ring_2Esemi_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Esemi_ring_2Esemi_ring\ A0) \quad (7)$$

Let $c_2Esemi_ring_2Erecordtype_2Esemi_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a \in (((((ty_2Esemi_ring_2Esemi_ring\ A_27a)^{(A_27a^{A_27a})^{A_27a}})^{(A_27a^{A_27a})^{A_27a}})^{A_27a})^{A_27a})^{A_27a} \quad (8)$$

Definition 7 We define $c_2Ering_2Esemi_ring_of$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ering_2Ering\ A_27a).(ap\ (ap\ c_2Ering_2Ering_R1\ V0r))$

Let $c_2Esemi_ring_2Esemi_ring_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR1\ A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (9)$$

Let $c_2Esemi_ring_2Esemi_ring_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR0\ A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (10)$$

Let $c_2Esemi_ring_2Esemi_ring_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRM\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (11)$$

Let $c_2Esemi_ring_2Esemi_ring_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRP\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring\ A_27a)}) \quad (12)$$

Definition 8 We define $c_2Esemi_ring_2Eis_semi_ring$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Esemi_ring_2Esemi_ring\ A_27a).(ap\ (ap\ c_2Esemi_ring_2Esemi_ring_SR1\ V0r))$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. (\forall V2z \in A_27a. (((V0x = V1y) \wedge (V1y = V2z)) \Rightarrow (V0x = V2z)))))) \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in A_27a. (\forall V1a0 \in A_27a. (\forall V2f \in ((A_27a^{A_27a})^{A_27a}). (\forall V3f0 \in ((A_27a^{A_27a})^{A_27a}). \\ & ((ap\ (c_2Esemi_ring_2Esemi_ring_SR0\ A_27a)\ (ap\ (ap\ (ap\ (ap\ (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V0a)\ V1a0)\ V2f)\ V3f0)) = V0a)))))) \wedge ((\forall V4a \in A_27a. (\forall V5a0 \in A_27a. \\ & (\forall V6f \in ((A_27a^{A_27a})^{A_27a}). (\forall V7f0 \in ((A_27a^{A_27a})^{A_27a}). \\ & ((ap\ (c_2Esemi_ring_2Esemi_ring_SR1\ A_27a)\ (ap\ (ap\ (ap\ (ap\ (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V4a)\ V5a0)\ V6f)\ V7f0)) = V5a0)))))) \wedge ((\forall V8a \in A_27a. (\forall V9a0 \in A_27a. \\ & (\forall V10f \in ((A_27a^{A_27a})^{A_27a}). (\forall V11f0 \in ((A_27a^{A_27a})^{A_27a}). \\ & ((ap\ (c_2Esemi_ring_2Esemi_ring_SRP\ A_27a)\ (ap\ (ap\ (ap\ (ap\ (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V8a)\ V9a0)\ V10f)\ V11f0)) = V10f)))))) \wedge ((\forall V12a \in A_27a. (\forall V13a0 \in \\ & A_27a. (\forall V14f \in ((A_27a^{A_27a})^{A_27a}). (\forall V15f0 \in ((A_27a^{A_27a})^{A_27a}). \\ & ((ap\ (c_2Esemi_ring_2Esemi_ring_SRM\ A_27a)\ (ap\ (ap\ (ap\ (ap\ (c_2Esemi_ring_2Erecordtype_2Esemi_ring\ A_27a)\ V12a)\ V13a0)\ V14f)\ V15f0)) = V15f0))))))))) \quad (17) \end{aligned}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (ty_2Ering_2Ering\ A_27a). ((p\ (ap\ (c_2Ering_2Eis_ring\ A_27a)\ V0r)) \Rightarrow (p\ (ap\ (c_2Esemi_ring_2Eis_semi_ring\ A_27a)\ (ap\ (c_2Ering_2Esemi_ring_of\ A_27a)\ V0r))))))$$