

thm_2Ering_2Ering_literal_11 (TMPBLUQR-
P Jp8myZWdHq4xD6AvdSVQ4dgA5n)

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Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (1)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Let $ty_2Ering_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ering_2Ering\ A0) \quad (2)$$

Let $c_2Ering_2Ering_R0_fupd : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R0_fupd\ A_27a \in ((ty_2Ering_2Ering\ A_27a)^{(ty_2Ering_2Ering\ A_27a)})^{(A_27a^{A_27a})} \quad (3)$$

Let $c_2Ering_2Ering_RN_fupd : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RN_fupd\ A_27a \in (((ty_2Ering_2Ering\ A_27a)^{(ty_2Ering_2Ering\ A_27a)})^{((A_27a^{A_27a})^{(A_27a^{A_27a})})}) \quad (4)$$

Let $c_2Ering_2Ering_RM_fupd : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RM_fupd\ A_27a \in (((ty_2Ering_2Ering\ A_27a)^{(ty_2Ering_2Ering\ A_27a)})^{((A_27a^{A_27a})^{A_27a})}^{((A_27a^{A_27a})^{A_27a})}) \quad (5)$$

Let $c_2Ering_2Ering_RP_fupd : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RP_fupd\ A_27a \in (((ty_2Ering_2Ering\ A_27a)^{(ty_2Ering_2Ering\ A_27a)})^{((A_27a^{A_27a})^{A_27a})}^{((A_27a^{A_27a})^{A_27a})}) \quad (6)$$

Let $c_2Ering_2Ering_R1_fupd : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R1_fupd\ A_27a \in ((ty_2Ering_2Ering\ A_27a)^{(ty_2Ering_2Ering\ A_27a)}(A_27a^{A_27a})) \quad (7)$$

Let $c_2Ering_2Ering_RN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RN\ A_27a \in ((A_27a^{A_27a})^{(ty_2Ering_2Ering\ A_27a)}) \quad (8)$$

Let $c_2Ering_2Ering_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RM\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Ering_2Ering\ A_27a)}) \quad (9)$$

Let $c_2Ering_2Ering_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_RP\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Ering_2Ering\ A_27a)}) \quad (10)$$

Let $c_2Ering_2Ering_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R1\ A_27a \in (A_27a^{(ty_2Ering_2Ering\ A_27a)}) \quad (11)$$

Let $c_2Ering_2Ering_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ering_2Ering_R0\ A_27a \in (A_27a^{(ty_2Ering_2Ering\ A_27a)}) \quad (12)$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V2t \in 2.V2t)))$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (ap\ (c_2Ecombin_2EK\ A_27a\ A_27b)\ V0x)\ V1y) = V0x))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0r1 \in (ty_2Ering_2Ering \\
& A.27a).(\forall V1r2 \in (ty_2Ering_2Ering\ A.27a).((V0r1 = V1r2) \Leftrightarrow \\
& (((ap\ (c_2Ering_2Ering_R0\ A.27a)\ V0r1) = (ap\ (c_2Ering_2Ering_R0 \\
& A.27a)\ V1r2)) \wedge (((ap\ (c_2Ering_2Ering_R1\ A.27a)\ V0r1) = (ap\ (c_2Ering_2Ering_R1 \\
& A.27a)\ V1r2)) \wedge (((ap\ (c_2Ering_2Ering_RP\ A.27a)\ V0r1) = (ap\ (c_2Ering_2Ering_RP \\
& A.27a)\ V1r2)) \wedge (((ap\ (c_2Ering_2Ering_RM\ A.27a)\ V0r1) = (ap\ (c_2Ering_2Ering_RM \\
& A.27a)\ V1r2)) \wedge (((ap\ (c_2Ering_2Ering_RN\ A.27a)\ V0r1) = (ap\ (c_2Ering_2Ering_RN \\
& A.27a)\ V1r2))))))))) \\
& \tag{15}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a01 \in A.27a.(\forall V1a1 \in \\
& A.27a.(\forall V2f11 \in ((A.27a^{A.27a})^{A.27a}).(\forall V3f01 \in (\\
& (A.27a^{A.27a})^{A.27a}).(\forall V4f1 \in (A.27a^{A.27a}).(\forall V5a02 \in \\
& A.27a.(\forall V6a2 \in A.27a.(\forall V7f12 \in ((A.27a^{A.27a})^{A.27a}). \\
& (\forall V8f02 \in ((A.27a^{A.27a})^{A.27a}).(\forall V9f2 \in (A.27a^{A.27a}). \\
& (((ap\ (ap\ (c_2Ering_2Ering_R0_fupd\ A.27a)\ (ap\ (c_2Ecombin_2EK \\
& A.27a\ A.27a)\ V0a01))\ (ap\ (ap\ (c_2Ering_2Ering_R1_fupd\ A.27a) \\
& (ap\ (c_2Ecombin_2EK\ A.27a\ A.27a)\ V1a1))\ (ap\ (ap\ (c_2Ering_2Ering_RP_fupd \\
& A.27a)\ (ap\ (c_2Ecombin_2EK\ ((A.27a^{A.27a})^{A.27a})\ ((A.27a^{A.27a})^{A.27a}) \\
& V2f11))\ (ap\ (ap\ (c_2Ering_2Ering_RM_fupd\ A.27a)\ (ap\ (c_2Ecombin_2EK \\
& ((A.27a^{A.27a})^{A.27a})\ ((A.27a^{A.27a})^{A.27a}))\ V3f01))\ (ap\ (ap\ (c_2Ering_2Ering_RN_fupd \\
& A.27a)\ (ap\ (c_2Ecombin_2EK\ (A.27a^{A.27a})\ (A.27a^{A.27a}))\ V4f1)) \\
& (c_2Ebool_2EARB\ (ty_2Ering_2Ering\ A.27a)))))) = (ap\ (ap\ (c_2Ering_2Ering_R0_fupd \\
& A.27a)\ (ap\ (c_2Ecombin_2EK\ A.27a\ A.27a)\ V5a02))\ (ap\ (ap\ (c_2Ering_2Ering_R1_fupd \\
& A.27a)\ (ap\ (c_2Ecombin_2EK\ A.27a\ A.27a)\ V6a2))\ (ap\ (ap\ (c_2Ering_2Ering_RP_fupd \\
& A.27a)\ (ap\ (c_2Ecombin_2EK\ ((A.27a^{A.27a})^{A.27a})\ ((A.27a^{A.27a})^{A.27a})) \\
& V7f12))\ (ap\ (ap\ (c_2Ering_2Ering_RM_fupd\ A.27a)\ (ap\ (c_2Ecombin_2EK \\
& ((A.27a^{A.27a})^{A.27a})\ ((A.27a^{A.27a})^{A.27a}))\ V8f02))\ (ap\ (ap\ (c_2Ering_2Ering_RN_fupd \\
& A.27a)\ (ap\ (c_2Ecombin_2EK\ (A.27a^{A.27a})\ (A.27a^{A.27a}))\ V9f2)) \\
& (c_2Ebool_2EARB\ (ty_2Ering_2Ering\ A.27a)))))) \Leftrightarrow ((V0a01 = V5a02) \wedge \\
& ((V1a1 = V6a2) \wedge ((V2f11 = V7f12) \wedge ((V3f01 = V8f02) \wedge (V4f1 = V9f2)))))))))
\end{aligned}$$