

thm_2Esemi_ring_2Emult_one_left
(TMbadYRqbFvngYcDTABLvciPrpi8Jmh8z6U)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Esemi_ring_2Esemi_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Esemi_ring_2Esemi_ring A0) \quad (1)$$

Let $c_2Esemi_ring_2Esemi_ring_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR1 A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring A_27a)}) \quad (2)$$

Let $c_2Esemi_ring_2Esemi_ring_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR0 A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring A_27a)}) \quad (3)$$

Let $c_2Esemi_ring_2Esemi_ring_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRM A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring A_27a)}) \quad (4)$$

Let $c_2Esemi_ring_2Esemi_ring_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRP A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring A_27a)}) \quad (5)$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 6 We define `c_2Esemi_ring_2Eis_semi_ring` to be $\lambda A_{27a} : \iota. \lambda V0r \in (ty_2Esemi_ring_2Esemi_ring$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0r \in (ty_2Esemi_ring_2Esemi_ring \\ & \quad A_{27a}). ((p\ (ap\ (c_2Esemi_ring_2Eis_semi_ring\ A_{27a})\ V0r)) \Rightarrow \\ & \quad (\forall V1n \in A_{27a}. ((ap\ (ap\ (ap\ (c_2Esemi_ring_2Esemi_ring_SRM \\ & \quad A_{27a})\ V0r)\ (ap\ (c_2Esemi_ring_2Esemi_ring_SR1\ A_{27a})\ V0r)) \\ & \quad \quad V1n) = V1n)))) \end{aligned}$$