

thm\_2Esemi\_ring\_2Emult\_sym  
(TMUNX6CzwVJ2FCMYB7FDZQKVU8MHR3MeMy9)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $ty\_2Esemi\_ring\_2Esemi\_ring : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Esemi\_ring\_2Esemi\_ring A0) \quad (1)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SR1 A\_27a \in (A\_27a^{(ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)}) \quad (2)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SR0 A\_27a \in (A\_27a^{(ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)}) \quad (3)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRM : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SRM A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)}) \quad (4)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SRP A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)}) \quad (5)$$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

**Definition 6** We define  $c\_2Esemi\_ring\_2Eis\_semi\_ring$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Esemi\_ring\_2Esemi\_ring$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Esemi\_ring\_2Esemi\_ring \\ & \quad A\_27a).((p\ (ap\ (c\_2Esemi\_ring\_2Eis\_semi\_ring\ A\_27a)\ V0r)) \Rightarrow \\ & (\forall V1n \in A\_27a.(\forall V2m \in A\_27a.((ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRM \\ & \quad A\_27a)\ V0r)\ V1n)\ V2m) = (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRM \\ & \quad A\_27a)\ V0r)\ V2m)\ V1n)))))) \end{aligned}$$