thm_2Esemi__ring_2Eplus__assoc (TMS5ukrs39btpAnMMAeBMJbwMzNL5Q599nq)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{1}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{2}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{3}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{4}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{5}$$

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Esemi__ring_2Eis__semi__ring *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Esemi\_\_ring\_2Esemi$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).((p\ (ap\ (\mathsf{c\_2Esemi\_\_ring\_2Eis\_\_semi\_\_ring}\ A\_27a)\ V0r)) \Rightarrow (\forall V1n \in A\_27a.(\forall V2m \in A\_27a.(\forall V3p \in A\_27a.((ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V0r)\ V1n)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V0r)\ V2m)\ V3p)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V0r)\ (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a)\ V0r)\ V1n)\ V2m))\ V3p)))))))$$