

thm_2Esemi_ring_2Eplus_zero_left (TMbyYC-cubkNLyHngAW7bHG3aDPdu8fqjXnF)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Let $ty_2Esemi_ring_2Esemi_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty_2Esemi_ring_2Esemi_ring \ A0) \quad (1)$$

Let $c_2Esemi_ring_2Esemi_ring_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR1 \ A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring \ A_27a)}) \quad (2)$$

Let $c_2Esemi_ring_2Esemi_ring_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SR0 \ A_27a \in (A_27a^{(ty_2Esemi_ring_2Esemi_ring \ A_27a)}) \quad (3)$$

Let $c_2Esemi_ring_2Esemi_ring_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRM \ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring \ A_27a)}) \quad (4)$$

Let $c_2Esemi_ring_2Esemi_ring_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Esemi_ring_2Esemi_ring_SRP \ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi_ring_2Esemi_ring \ A_27a)}) \quad (5)$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_21$ to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap \ (ap \ (c_2Emin_2E_3D \ (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (c_2Ebool_2E_21 \ 2)) \ (\lambda V2t \in 2.V2t)))$

Definition 6 We define `c_2Esemi_ring_2Eis_semi_ring` to be $\lambda A_{27a} : \iota. \lambda V0r \in (ty_2Esemi_ring_2Esemi_ring$

Theorem 1

$$\begin{aligned} \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0r \in (ty_2Esemi_ring_2Esemi_ring \\ A_{27a}). (p\ (ap\ (c_2Esemi_ring_2Eis_semi_ring\ A_{27a})\ V0r)) \Rightarrow \\ (\forall V1n \in A_{27a}. (ap\ (ap\ (ap\ (c_2Esemi_ring_2Esemi_ring_SRP \\ A_{27a})\ V0r)\ (ap\ (c_2Esemi_ring_2Esemi_ring_SR0\ A_{27a})\ V0r)) \\ V1n) = V1n)))) \end{aligned}$$