

thm\_2Esemi\_ring\_2Esemi\_ring\_fn\_updates  
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LokCxLVHDRya5XF6nnjwPCCcR7kUXdKQD)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Esemi\_ring\_2Esemi\_ring : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Esemi\_ring\_2Esemi\_ring A0) \quad (1)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR0\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SR0\_fupd \\ & A\_27a \in (((ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)^{(ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)})^{(A\_27a^{A\_27a})}) \end{aligned} \quad (2)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR1\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SR1\_fupd \\ & A\_27a \in (((ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)^{(ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)})^{(A\_27a^{A\_27a})}) \end{aligned} \quad (3)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRP\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SRP\_fupd \\ & A\_27a \in (((ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)^{(ty\_2Esemi\_ring\_2Esemi\_ring A\_27a)})^{(((A\_27a^{A\_27a})^{A\_27a})^{A\_27a})}) \end{aligned} \quad (4)$$

Let  $c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a \in (((((ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)((A\_27a^{A\_27a})^{A\_27a}))((A\_27a^{A\_27a})^{A\_27a}))A\_27a)A\_27a) \end{aligned} \quad (5)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRM\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SRM\_fupd \\ & A\_27a \in ((ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)(ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a))((A\_27a^{A\_27a})^{A\_27a}) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f1 \in (A\_27a^{A\_27a}).(\forall V1a \in \\ & A\_27a.(\forall V2a0 \in A\_27a.(\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\ & (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR0\_fupd \\ & A\_27a)\ V0f1)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0))) = (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a)\ (ap\ V0f1\ V1a))\ V2a0)\ V3f)\ V4f0)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f1 \in (A\_27a^{A\_27a}).(\forall V1a \in \\ & A\_27a.(\forall V2a0 \in A\_27a.(\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\ & (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR1\_fupd \\ & A\_27a)\ V0f1)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0))) = (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a)\ V1a)\ (ap\ V0f1\ V2a0))\ V3f)\ V4f0)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f1 \in (((A\_27a^{A\_27a})^{A\_27a})((A\_27a^{A\_27a})^{A\_27a})). \\ & (\forall V1a \in A\_27a.(\forall V2a0 \in A\_27a.(\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\ & (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRP\_fupd \\ & A\_27a)\ V0f1)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0))) = (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a)\ V1a)\ V2a0)\ (ap\ V0f1\ V3f))\ V4f0)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f1 \in (((A\_27a^{A\_27a})^{A\_27a})((A\_27a^{A\_27a})^{A\_27a})). \\ & (\forall V1a \in A\_27a.(\forall V2a0 \in A\_27a.(\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\ & (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}).((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRM\_fupd \\ & A\_27a)\ V0f1)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0))) = (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\ & A\_27a)\ V1a)\ V2a0)\ V3f)\ (ap\ V0f1\ V4f0)))))) \end{aligned} \quad (10)$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f1 \in (A\_27a^{A\_27a}). ( \\
& \quad \forall V1a \in A\_27a. (\forall V2a0 \in A\_27a. (\forall V3f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad (\forall V4f0 \in ((A\_27a^{A\_27a})^{A\_27a}). ((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR0\_fupd \\
& \quad \quad A\_27a)\ V0f1)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \quad \quad A\_27a)\ V1a)\ V2a0)\ V3f)\ V4f0))) = (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \quad \quad A\_27a)\ (ap\ V0f1\ V1a))\ V2a0)\ V3f)\ V4f0)))))) \wedge ((\forall V5f1 \in (A\_27a^{A\_27a}). \\
& \quad (\forall V6a \in A\_27a. (\forall V7a0 \in A\_27a. (\forall V8f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad (\forall V9f0 \in ((A\_27a^{A\_27a})^{A\_27a}). ((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR1\_fupd \\
& \quad \quad A\_27a)\ V5f1)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \quad \quad A\_27a)\ V6a)\ V7a0)\ V8f)\ V9f0))) = (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \quad \quad A\_27a)\ V6a)\ (ap\ V5f1\ V7a0))\ V8f)\ V9f0)))))) \wedge ((\forall V10f1 \in ( \\
& \quad ((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{A\_27a})^{A\_27a}}). (\forall V11a \in \\
& \quad A\_27a. (\forall V12a0 \in A\_27a. (\forall V13f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad (\forall V14f0 \in ((A\_27a^{A\_27a})^{A\_27a}). ((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRP\_fupd \\
& \quad \quad A\_27a)\ V10f1)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \quad \quad A\_27a)\ V11a)\ V12a0)\ V13f)\ V14f0))) = (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \quad \quad A\_27a)\ V11a)\ V12a0)\ (ap\ V10f1\ V13f))\ V14f0)))))) \wedge (\forall V15f1 \in \\
& \quad (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{A\_27a})^{A\_27a}}). (\forall V16a \in \\
& \quad A\_27a. (\forall V17a0 \in A\_27a. (\forall V18f \in ((A\_27a^{A\_27a})^{A\_27a}). \\
& \quad (\forall V19f0 \in ((A\_27a^{A\_27a})^{A\_27a}). ((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRM\_fupd \\
& \quad \quad A\_27a)\ V15f1)\ (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \quad \quad A\_27a)\ V16a)\ V17a0)\ V18f)\ V19f0))) = (ap\ (ap\ (ap\ (ap\ (c\_2Esemi\_ring\_2Erecordtype\_2Esemi\_ring \\
& \quad \quad A\_27a)\ V16a)\ V17a0)\ V18f)\ (ap\ V15f1\ V19f0)))))))))
\end{aligned}$$