

thm\_2Esemi\_ring\_2Esemi\_ring\_literal\_11  
(TMQLqEzuFLRuEXsLBfnx4317ff47Sv3KRbz)

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Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (1)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Let  $ty\_2Esemi\_ring\_2Esemi\_ring : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_ring\_2Esemi\_ring\ A0) \quad (2)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR0\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SR0\_fupd\ A\_27a \in (((ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)})(A\_27a^{A\_27a})) \quad (3)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRM\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SRM\_fupd\ A\_27a \in (((ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)})(A\_27a^{A\_27a})^{A\_27a}) \quad (4)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRP\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_ring\_2Esemi\_ring\_SRP\_fupd\ A\_27a \in (((ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A\_27a)})(A\_27a^{A\_27a})^{A\_27a}) \quad (5)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR1\_fupd : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c.2Esemi\_ring\_2Esemi\_ring\_SR1\_fupd \\ & A.27a \in (((ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a)^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a)})^{(A.27a^{A.27a})}) \end{aligned} \quad (6)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRM : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c.2Esemi\_ring\_2Esemi\_ring\_SRM \\ & A.27a \in (((A.27a^{A.27a})^{A.27a})^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a)}) \end{aligned} \quad (7)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SRP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c.2Esemi\_ring\_2Esemi\_ring\_SRP \\ & A.27a \in (((A.27a^{A.27a})^{A.27a})^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a)}) \end{aligned} \quad (8)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c.2Esemi\_ring\_2Esemi\_ring\_SR1 \\ & A.27a \in (A.27a^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a)}) \end{aligned} \quad (9)$$

Let  $c\_2Esemi\_ring\_2Esemi\_ring\_SR0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c.2Esemi\_ring\_2Esemi\_ring\_SR0 \\ & A.27a \in (A.27a^{(ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a)}) \end{aligned} \quad (10)$$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A.27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (ap\ (c\_2Ecombin\_2EK\ A.27a\ A.27b)\ V0x)\ V1y) = V0x))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0s \in (ty.2Esemi\_ring.2Esemi\_ring \\
& A.27a).(\forall V1f \in (A.27a^{A.27a}).((ap (c.2Esemi\_ring.2Esemi\_ring\_SR0 \\
& A.27a) (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SR1\_fupd A.27a) \\
& V1f) V0s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SR0 A.27a) V0s)))) \wedge \\
& ((\forall V2s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V3f \in \\
& (((A.27a^{A.27a})^{A.27a})^{(A.27a^{A.27a})^{A.27a}})).((ap (c.2Esemi\_ring.2Esemi\_ring\_SR0 \\
& A.27a) (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SRP\_fupd A.27a) \\
& V3f) V2s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SR0 A.27a) V2s)))) \wedge \\
& ((\forall V4s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V5f \in \\
& (((A.27a^{A.27a})^{A.27a})^{(A.27a^{A.27a})^{A.27a}})).((ap (c.2Esemi\_ring.2Esemi\_ring\_SR0 \\
& A.27a) (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SRM\_fupd A.27a) \\
& V5f) V4s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SR0 A.27a) V4s)))) \wedge \\
& ((\forall V6s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V7f \in \\
& (A.27a^{A.27a}).((ap (c.2Esemi\_ring.2Esemi\_ring\_SR1 A.27a) \\
& (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SR0\_fupd A.27a) V7f) \\
& V6s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SR1 A.27a) V6s)))) \wedge \\
& ((\forall V8s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V9f \in \\
& (((A.27a^{A.27a})^{A.27a})^{(A.27a^{A.27a})^{A.27a}})).((ap (c.2Esemi\_ring.2Esemi\_ring\_SR1 \\
& A.27a) (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SRP\_fupd A.27a) \\
& V9f) V8s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SR1 A.27a) V8s)))) \wedge \\
& ((\forall V10s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V11f \in \\
& (((A.27a^{A.27a})^{A.27a})^{(A.27a^{A.27a})^{A.27a}})).((ap (c.2Esemi\_ring.2Esemi\_ring\_SR1 \\
& A.27a) (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SRM\_fupd A.27a) \\
& V11f) V10s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SR1 A.27a) V10s)))) \wedge \\
& ((\forall V12s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V13f \in \\
& (A.27a^{A.27a}).((ap (c.2Esemi\_ring.2Esemi\_ring\_SRP A.27a) \\
& (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SR0\_fupd A.27a) V13f) \\
& V12s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SRP A.27a) V12s)))) \wedge \\
& ((\forall V14s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V15f \in \\
& (A.27a^{A.27a}).((ap (c.2Esemi\_ring.2Esemi\_ring\_SRP A.27a) \\
& (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SR1\_fupd A.27a) V15f) \\
& V14s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SRP A.27a) V14s)))) \wedge \\
& ((\forall V16s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V17f \in \\
& (((A.27a^{A.27a})^{A.27a})^{(A.27a^{A.27a})^{A.27a}})).((ap (c.2Esemi\_ring.2Esemi\_ring\_SRP \\
& A.27a) (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SRM\_fupd A.27a) \\
& V17f) V16s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SRP A.27a) V16s)))) \wedge \\
& ((\forall V18s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V19f \in \\
& (A.27a^{A.27a}).((ap (c.2Esemi\_ring.2Esemi\_ring\_SRM A.27a) \\
& (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SR0\_fupd A.27a) V19f) \\
& V18s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SRM A.27a) V18s)))) \wedge \\
& ((\forall V20s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V21f \in \\
& (A.27a^{A.27a}).((ap (c.2Esemi\_ring.2Esemi\_ring\_SRM A.27a) \\
& (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SR1\_fupd A.27a) V21f) \\
& V20s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SRM A.27a) V20s)))) \wedge \\
& ((\forall V22s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V23f \in \\
& (((A.27a^{A.27a})^{A.27a})^{(A.27a^{A.27a})^{A.27a}})).((ap (c.2Esemi\_ring.2Esemi\_ring\_SRM \\
& A.27a) (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SRP\_fupd A.27a) \\
& V23f) V22s)) = (ap (c.2Esemi\_ring.2Esemi\_ring\_SRM A.27a) V22s)))) \wedge \\
& ((\forall V24s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a).(\forall V25f \in \\
& (A.27a^{A.27a}).((ap (c.2Esemi\_ring.2Esemi\_ring\_SR0 A.27a) \\
& (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SR0\_fupd A.27a) V25f) \\
& V24s)) = (ap V25f (ap (c.2Esemi\_ring.2Esemi\_ring\_SR0 A.27a) \\
& V24s)))))) \wedge ((\forall V26s \in (ty.2Esemi\_ring.2Esemi\_ring A.27a). \\
& (\forall V27f \in (A.27a^{A.27a}).((ap (c.2Esemi\_ring.2Esemi\_ring\_SR1 \\
& A.27a) (ap (ap (c.2Esemi\_ring.2Esemi\_ring\_SR1\_fupd A.27a) \\
& V27f) V26s)) = (ap V27f (ap (c.2Esemi\_ring.2Esemi\_ring\_SR1 \\
& A.27a) V26s)))))) \wedge ((\forall V28s \in (ty.2Esemi\_ring.2Esemi\_ring
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s1 \in (ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a). (\forall V1s2 \in (ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a). \\
& ((V0s1 = V1s2) \Leftrightarrow (((ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR0\ A.27a)\ V0s1) = (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR0\ A.27a)\ V1s2)) \wedge \\
& (((ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR1\ A.27a)\ V0s1) = (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR1\ A.27a)\ V1s2)) \wedge (((ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRP\ A.27a)\ V0s1) = (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRP\ A.27a)\ V1s2)) \wedge \\
& (((ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRM\ A.27a)\ V0s1) = (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRM\ A.27a)\ V1s2)))))))))
\end{aligned} \tag{13}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a01 \in A.27a. (\forall V1a1 \in A.27a. (\forall V2f01 \in ((A.27a^{A.27a})^{A.27a}). (\forall V3f1 \in ((A.27a^{A.27a})^{A.27a}). (\forall V4a02 \in A.27a. (\forall V5a2 \in A.27a. \\
& (\forall V6f02 \in ((A.27a^{A.27a})^{A.27a}). (\forall V7f2 \in ((A.27a^{A.27a})^{A.27a}). \\
& (((ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR0\_fupd\ A.27a)\ (ap\ (c\_2Ecombin\_2EK\ A.27a\ A.27a)\ V0a01))\ (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR1\_fupd\ A.27a)\ (ap\ (c\_2Ecombin\_2EK\ A.27a\ A.27a)\ V1a1))\ (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRP\_fupd\ A.27a)\ (ap\ (c\_2Ecombin\_2EK\ ((A.27a^{A.27a})^{A.27a})\ ((A.27a^{A.27a})^{A.27a}))\ V2f01))\ (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRM\_fupd\ A.27a)\ (ap\ (c\_2Ecombin\_2EK\ ((A.27a^{A.27a})^{A.27a})\ ((A.27a^{A.27a})^{A.27a}))\ V3f1))\ (c\_2Ebool\_2EARB\ (ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a)))))) = \\
& (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR0\_fupd\ A.27a)\ (ap\ (c\_2Ecombin\_2EK\ A.27a\ A.27a)\ V4a02))\ (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SR1\_fupd\ A.27a)\ (ap\ (c\_2Ecombin\_2EK\ A.27a\ A.27a)\ V5a2))\ (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRP\_fupd\ A.27a)\ (ap\ (c\_2Ecombin\_2EK\ ((A.27a^{A.27a})^{A.27a})\ ((A.27a^{A.27a})^{A.27a}))\ V6f02))\ (ap\ (ap\ (c\_2Esemi\_ring\_2Esemi\_ring\_SRM\_fupd\ A.27a)\ (ap\ (c\_2Ecombin\_2EK\ ((A.27a^{A.27a})^{A.27a})\ ((A.27a^{A.27a})^{A.27a}))\ V7f2))\ (c\_2Ebool\_2EARB\ (ty\_2Esemi\_ring\_2Esemi\_ring\ A.27a)))))) \Leftrightarrow \\
& ((V0a01 = V4a02) \wedge ((V1a1 = V5a2) \wedge ((V2f01 = V6f02) \wedge (V3f1 = V7f2))))))
\end{aligned}$$