

thm_2Eseq_2ESEQ
(TMVUyHkQ6nd2F84Uq28QszfywhXaGsgzFq5)

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Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \tag{3}$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})) \tag{4}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ V0P))))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{6}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{7}$$

Definition 4 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (8)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (9)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (10)$$

Definition 5 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))))$

Definition 7 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E_40\ (ty_2Erealax_2Ereal)))$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (11)$$

Definition 8 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (12)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (14)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 11 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. (ap\ c_2Erealax_2Ereal)$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}}_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \quad (15)$$

Definition 12 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 13 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 14 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E2F)\ (\lambda V0t \in 2.V0t))$.

Definition 15 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 16 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 17 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E2F)\ (\lambda V2t \in 2.V2t))))$

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Ebool_2ECOND)\ t1\ t2))))$

Definition 19 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND)\ x)\ t)\ t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 20 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Definition 21 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 22 We define $c_2Earithmic_2E3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E2F)\ (\lambda V2t \in 2.V2t))))$

Definition 24 We define $c_2Earithmic_2E3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (18)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0d \in (ty.2Emetric.2Emetric \\
& \quad A.27a).(\forall V1x \in (A.27a^{ty.2Enum.2Enum}).(\forall V2x0 \in A.27a. \\
& \quad ((p\ (ap\ (ap\ (ap\ (c.2Enets.2Etends\ A.27a\ ty.2Enum.2Enum)\ V1x)\ V2x0) \\
& \quad (ap\ (ap\ (c.2Epair.2E.2C\ (ty.2Etopology.2Etopology\ A.27a)\ ((2^{ty.2Enum.2Enum})^{ty.2Enum.2Enum})) \\
& \quad (ap\ (c.2Emetric.2Emtop\ A.27a)\ V0d))\ c.2Earithmetic.2E.3E.3D))) \Leftrightarrow \\
& \quad (\forall V3e \in ty.2Erealax.2Ereal.((p\ (ap\ (ap\ c.2Erealax.2Ereal_lt \\
& \quad (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ V3e)) \Rightarrow (\exists V4N \in \\
& \quad ty.2Enum.2Enum.(\forall V5n \in ty.2Enum.2Enum.((p\ (ap\ (ap\ c.2Earithmetic.2E.3E.3D \\
& \quad V5n)\ V4N)) \Rightarrow (p\ (ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ (ap\ (c.2Emetric.2Edist \\
& \quad A.27a)\ V0d)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27a)\ (ap\ V1x\ V5n))\ V2x0))) \\
& \quad V3e)))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealax.2Ereal.(\forall V1y \in ty.2Erealax.2Ereal. \\
& \quad ((ap\ c.2Ereal.2Eabs\ (ap\ (ap\ c.2Ereal.2Ereal_sub\ V0x)\ V1y)) = (\\
& \quad ap\ c.2Ereal.2Eabs\ (ap\ (ap\ c.2Ereal.2Ereal_sub\ V1y)\ V0x))))
\end{aligned} \tag{27}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in (ty.2Erealax.2Ereal^{ty.2Enum.2Enum}).(\forall V1x0 \in \\
& \quad ty.2Erealax.2Ereal.((p\ (ap\ (ap\ c.2Eseq.2E.2D.2D.3E\ V0x)\ V1x0)) \Leftrightarrow \\
& \quad (\forall V2e \in ty.2Erealax.2Ereal.((p\ (ap\ (ap\ c.2Erealax.2Ereal_lt \\
& \quad (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ V2e)) \Rightarrow (\exists V3N \in \\
& \quad ty.2Enum.2Enum.(\forall V4n \in ty.2Enum.2Enum.((p\ (ap\ (ap\ c.2Earithmetic.2E.3E.3D \\
& \quad V4n)\ V3N)) \Rightarrow (p\ (ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal.2Eabs \\
& \quad (ap\ (ap\ c.2Ereal.2Ereal_sub\ (ap\ V0x\ V4n))\ V1x0)))\ V2e)))))))))
\end{aligned}$$