

thm_2Eseq_2ESEQ__NEG__BOUNDED
(TMYKpz1AbTgRQVEMniKkyhoi5xtqaEGUmm2)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \tag{5}$$

Definition 16 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECONI$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Definition 18 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 19 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 22 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (15)$$

Definition 23 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 24 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (16)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (17)$$

Definition 25 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a \cdot 27b})$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (18)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric A_27a)^{(ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod A_27a A_27a)}})) \quad (19)$$

Definition 26 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealx_2Ereal) (ap (c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (20)$$

Definition 27 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Enets_2Ebunded : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Ebunded \\ A_27a A_27b \in ((2^{(A_27a)^{A_27b}})(ty_2Epair_2Eprod (ty_2Emetric_2Emetric A_27a) ((2^{A_27b})^{A_27a}))) \end{aligned} \quad (21)$$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Erealx_2Ereal. ((ap c_2Ereal_2Eabs (ap c_2Erealx_2Ereal_neg \\ V0x)) = (ap c_2Ereal_2Eabs V0x))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0s \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}). ((p (ap (ap \\ (c_2Enets_2Ebunded ty_2Erealx_2Ereal ty_2Enum_2Enum) (ap \\ (ap (c_2Epair_2E_2C (ty_2Emetric_2Emetric ty_2Erealx_2Ereal) \\ ((2^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})) c_2Emetric_2Emr1 c_2Earithmetic_2E_3E_3D)) \\ V0s)) \Leftrightarrow (\exists V1k \in ty_2Erealx_2Ereal. (\forall V2n \in ty_2Enum_2Enum. \\ (p (ap (ap c_2Erealx_2Ereal_lt (ap c_2Ereal_2Eabs (ap V0s V2n))) \\ V1k)))))) \end{aligned} \quad (25)$$

Theorem 1

$$\begin{aligned} (\forall V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}). ((p (ap (ap \\ (c_2Enets_2Ebunded ty_2Erealx_2Ereal ty_2Enum_2Enum) (ap \\ (ap (c_2Epair_2E_2C (ty_2Emetric_2Emetric ty_2Erealx_2Ereal) \\ ((2^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})) c_2Emetric_2Emr1 c_2Earithmetic_2E_3E_3D)) \\ (\lambda V1n \in ty_2Enum_2Enum. (ap c_2Erealx_2Ereal_neg (ap V0f \\ V1n)))))) \Leftrightarrow (p (ap (ap (c_2Enets_2Ebunded ty_2Erealx_2Ereal ty_2Enum_2Enum) \\ (ap (ap (c_2Epair_2E_2C (ty_2Emetric_2Emetric ty_2Erealx_2Ereal) \\ ((2^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})) c_2Emetric_2Emr1 c_2Earithmetic_2E_3E_3D)) \\ V0f)))) \end{aligned}$$