

thm_2Eseq_2ESEQ__POWER
(TMRngjo15G4rmK1Ev6Aiswgog7vrdA81zMM)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{4}$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealx_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealx_2Ereal}) \tag{5}$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1Q \in 2.V1Q))\ (\lambda V2R \in 2.V2R))\ P\ Q)$.

Definition 5 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$.

Definition 8 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21) 2) (\lambda V2t \in 2)))$.
Let $c_EEnum_E_REP_num : \iota$ be given. Assume the following.

$$c_EEnum_E_REP_num \in (\omega^{ty_EEnum_E_Enum}) \quad (6)$$

Let $c_EEnum_E_ESUC_REP : \iota$ be given. Assume the following.

$$c_EEnum_E_ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Definition 9 We define $c_EEnum_E_ESUC$ to be $\lambda V0m \in ty_EEnum_E_Enum.(ap c_EEnum_E_EABS_num)$

Definition 10 We define $c_Emin_E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_Ebool_E_3F$ to be $\lambda A^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_Emin_E_40) P)))$

Definition 12 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_EEnum_E_Enum.\lambda V1n \in ty_EEnum_E_Enum$

Definition 13 We define $c_Earithmic_E_3E$ to be $\lambda V0m \in ty_EEnum_E_Enum.\lambda V1n \in ty_EEnum_E_Enum$

Definition 14 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21) 2) (\lambda V2t \in 2)))$

Definition 15 We define $c_Earithmic_E_3E_3D$ to be $\lambda V0m \in ty_EEnum_E_Enum.\lambda V1n \in ty_EEnum_E_Enum$

Let $ty_Ehreal_E_hreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_E_hreal \quad (8)$$

Let $ty_Epair_E_Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_Epair_E_Eprod\ A0\ A1) \quad (9)$$

Let $c_Erealax_E_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_E_Ereal_REP_CLASS \in ((2^{(ty_Epair_E_Eprod\ ty_Ehreal_E_hreal\ ty_Ehreal_E_hreal)})^{ty_Erealax_E_Ereal}) \quad (10)$$

Definition 16 We define $c_Erealax_E_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_E_Ereal.(ap (c_Emin_E_40) (the (lambda x.x \in A \wedge P x)))$

Let $c_Erealax_E_Etreal_neg : \iota$ be given. Assume the following.

$$c_Erealax_E_Etreal_neg \in ((ty_Epair_E_Eprod\ ty_Ehreal_E_hreal\ ty_Ehreal_E_hreal)^{ty_Epair_E_Eprod\ ty_Ehreal_E_hreal\ ty_Ehreal_E_hreal}) \quad (11)$$

Let $c_Erealax_E_Etreal_eq : \iota$ be given. Assume the following.

$$c_Erealax_E_Etreal_eq \in ((2^{(ty_Epair_E_Eprod\ ty_Ehreal_E_hreal\ ty_Ehreal_E_hreal)})^{ty_Epair_E_Eprod\ ty_Ehreal_E_hreal\ ty_Ehreal_E_hreal}) \quad (12)$$

Let $c_Erealax_E_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_E_Ereal_ABS_CLASS \in (ty_Erealax_E_Ereal)^{(2^{(ty_Epair_E_Eprod\ ty_Ehreal_E_hreal\ ty_Ehreal_E_hreal)})^{ty_Epair_E_Eprod\ ty_Ehreal_E_hreal\ ty_Ehreal_E_hreal}} \quad (13)$$

Definition 17 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 18 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (14)$$

Definition 19 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 20 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (15)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (16)$$

Definition 21 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 22 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 24 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (17)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (18)$$

Definition 25 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (19)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (20)$$

Definition 26 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealx_2Ereal) (ap (c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (21)$$

Definition 27 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod A_27a A_27b)})^{(2^{A_27b})^{A_27a}}) \quad (22)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (23)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (24)$$

Definition 28 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric A_27a). (ap (c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a)^{A_27b}}) \quad (25)$$

Definition 29 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}). \lambda V1x$

Definition 30 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (26)$$

Definition 31 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic$

Definition 32 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Eabs V0c)) V1n) = (ap c_2Ereal_2Eabs \\
& (ap (ap c_2Ereal_2Epow V0c) V1n))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). ((p (ap (ap \\
& c_2Eseq_2E_2D_2D_3E (\lambda V1n \in ty_2Enum_2Enum. (ap c_2Ereal_2Eabs \\
& (ap V0f V1n)))) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \Leftrightarrow \\
& (p (ap (ap c_2Eseq_2E_2D_2D_3E V0f) (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Eabs V0c)) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Rightarrow (p (ap \\
& (ap c_2Eseq_2E_2D_2D_3E (\lambda V1n \in ty_2Enum_2Enum. (ap (ap c_2Ereal_2Epow \\
& (ap c_2Ereal_2Eabs V0c)) V1n))) (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))))
\end{aligned} \tag{30}$$

Theorem 1

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Eabs V0c)) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Rightarrow (p (ap \\
& (ap c_2Eseq_2E_2D_2D_3E (\lambda V1n \in ty_2Enum_2Enum. (ap (ap c_2Ereal_2Epow \\
& V0c) V1n))) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned}$$