

thm\_2Eseq\_2ESER\_\_POS\_\_LT  
(TMKV47sH3WbTfi8DrRYgsanskB2E8hyhphQ)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_Earithmic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmic\_EBIT1 V0n))$ .

**Definition 8** We define  $c\_Earithmic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_Ebool\_EF$  to be  $(ap (c\_Ebool\_E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_Emin\_E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_Ebool\_E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_E3D\_3D\_3E V0t) c\_Ebool\_E21))$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (7)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (8)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (10)$$

**Definition 12** We define  $c\_Emin\_E40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_Emin\_E40 V0a))$ .

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etreall\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (12)$$

Let  $c\_2Erealax\_2Etreall\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (14)$$

**Definition 15** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 17** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21\ 2) (\lambda V2t \in$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (15)$$

**Definition 18** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum})})^{(ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum)} \quad (16)$$

**Definition 19** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P (ap (c\_2Emin\_2E\_40$

**Definition 20** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

**Definition 21** We define  $c\_2Earithmic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

**Definition 22** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21\ 2) (\lambda V2t \in$

**Definition 23** We define  $c\_2Earithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (17)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 25** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (18)$$

**Definition 26** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 27** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.$

**Definition 28** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECOND$

Let  $c\_2\text{Epair\_2ESND} : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2\text{Epair\_2ESND} \\ A\_27a \ A\_27b \in (A\_27b)^{(ty\_2\text{Epair\_2Eprod } A\_27a \ A\_27b)}} \end{aligned} \quad (19)$$

Let  $c\_2\text{Epair\_2EFST} : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2\text{Epair\_2EFST} \\ A\_27a \ A\_27b \in (A\_27a)^{(ty\_2\text{Epair\_2Eprod } A\_27a \ A\_27b)}} \end{aligned} \quad (20)$$

**Definition 29** We define  $c\_2\text{Epair\_2EUNCURRY}$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c)^{A\_27a})$

Let  $ty\_2\text{Emetric\_2Emetric} : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty\_2\text{Emetric\_2Emetric } A0) \quad (21)$$

Let  $c\_2\text{Emetric\_2Emetric} : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2\text{Emetric\_2Emetric } A\_27a \in ((ty\_2\text{Emetric\_2Emetric} \\ A\_27a)^{(ty\_2\text{Erealax\_2Ereal}^{(ty\_2\text{Epair\_2Eprod } A\_27a \ A\_27a)}})) \end{aligned} \quad (22)$$

**Definition 30** We define  $c\_2\text{Emetric\_2Emr1}$  to be  $(\text{ap } (c\_2\text{Emetric\_2Emetric } ty\_2\text{Erealax\_2Ereal}) (\text{ap } (c\_2\text{Emetric\_2Emetric } ty\_2\text{Erealax\_2Ereal}^{(ty\_2\text{Epair\_2Eprod } A\_27a \ A\_27a)}))$

Let  $c\_2\text{Emetric\_2Edist} : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2\text{Emetric\_2Edist } A\_27a \in ((ty\_2\text{Erealax\_2Ereal}^{(ty\_2\text{Epair\_2Eprod } A\_27a \ A\_27a)})) \quad (23)$$

Let  $ty\_2\text{Etopology\_2Etopology} : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty\_2\text{Etopology\_2Etopology } A0) \quad (24)$$

Let  $c\_2\text{Etopology\_2Etopology} : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2\text{Etopology\_2Etopology } A\_27a \in \\ ((ty\_2\text{Etopology\_2Etopology } A\_27a)^{(2^{(2^{A\_27a})}})) \end{aligned} \quad (25)$$

**Definition 31** We define  $c\_2\text{Emetric\_2Emtop}$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2\text{Emetric\_2Emetric } A\_27a). (\text{ap } (c\_2\text{Emetric\_2Emetric } ty\_2\text{Erealax\_2Ereal}^{(ty\_2\text{Epair\_2Eprod } A\_27a \ A\_27a)}))$

Let  $c\_2\text{Enets\_2Etends} : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2\text{Enets\_2Etends} \\ A\_27a \ A\_27b \in (((2^{(ty\_2\text{Epair\_2Eprod } (ty\_2\text{Etopology\_2Etopology } A\_27a) ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a)^{A\_27b}})) \end{aligned} \quad (26)$$

**Definition 32** We define  $c\_2\text{Eseq\_2E\_2D\_2D\_3E}$  to be  $\lambda V0x \in (ty\_2\text{Erealax\_2Ereal}^{ty\_2\text{Enum\_2Enum}}). \lambda V1x \in (ty\_2\text{Erealax\_2Ereal}^{ty\_2\text{Enum\_2Enum}})$

**Definition 33** We define  $c\_2\text{Eseq\_2Esums}$  to be  $\lambda V0f \in (ty\_2\text{Erealax\_2Ereal}^{ty\_2\text{Enum\_2Enum}}). \lambda V1s \in (ty\_2\text{Erealax\_2Ereal}^{ty\_2\text{Enum\_2Enum}})$

**Definition 34** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap (c\_2E$

**Definition 35** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 36** We define  $c\_2Eseq\_2Esummable$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap (c$

Assume the following.

$$\begin{aligned} & ((ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)) = \\ & \quad (ap c\_2Enum\_2ESUC c\_2Enum\_2E0)) \end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & \quad ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ & \quad ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\ & \quad (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ & \quad V0m) V1n)))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & \quad V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))))) \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & \quad V0m) (ap c\_2Enum\_2ESUC V0m)))) \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC V0m) = (ap (ap \\ & \quad c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ & \quad c\_2Earithmetic\_2EZERO)))))) \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & \quad \forall V2p \in ty\_2Enum\_2Enum.(((p (ap c\_2Earithmetic\_2E\_3C\_3D \\ & \quad V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\ & \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & \quad V0m) V0m))) \end{aligned} \tag{32}$$

Assume the following.

$$True \tag{33}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & \quad (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_add\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x) = V0x)) \quad (37)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ V0x)\ V1y)))))) \quad (38)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V0x)\ V2z)))))) \quad (39)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V0x)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V1y)))))) \quad (40)$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum. (\forall V1f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). ((ap\ (ap\ c\_2Ereal\_2Esum\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V0n)\ c\_2Enum\_2E0))\ V1f) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3m \in ty\_2Enum\_2Enum. (\forall V4f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). ((ap\ (ap\ c\_2Ereal\_2Esum\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V2n)\ (ap\ c\_2Enum\_2ESUC\ V3m)))\ V4f) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ (ap\ (ap\ c\_2Ereal\_2Esum\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V2n)\ V3m))\ V4f))\ (ap\ V4f\ (ap\ (ap\ c\_2Earithmic\_2E\_2B\ V2n)\ V3m)))))))))) \quad (41)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1n \in \\
& ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
& ty\_2Enum\_2Enum) c\_2Enum\_2E0) V1n)) V0f)) (ap (ap c\_2Ereal\_2Esum \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) V1n) V2p)) \\
& V0f)) = (ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
& ty\_2Enum\_2Enum) c\_2Enum\_2E0) (ap (ap c\_2Earithmetic\_2E\_2B V1n) \\
& V2p)))) V0f))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1n \in \\
& ty\_2Enum\_2Enum.(((p (ap c\_2Eseq\_2Esummable V0f)) \wedge (\forall V2m \in \\
& ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2m)) \Rightarrow \\
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap V0f V2m)))))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
& ty\_2Enum\_2Enum) c\_2Enum\_2E0) V1n)) V0f)) (ap c\_2Eseq\_2Esuminf \\
& V0f))))))
\end{aligned} \tag{43}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1n \in \\
& ty\_2Enum\_2Enum.(((p (ap c\_2Eseq\_2Esummable V0f)) \wedge (\forall V2m \in \\
& ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2m)) \Rightarrow \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap V0f V2m)))))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
& ty\_2Enum\_2Enum) c\_2Enum\_2E0) V1n)) V0f)) (ap c\_2Eseq\_2Esuminf \\
& V0f))))))
\end{aligned}$$