

thm_2Eseq_2ESUMINF__2D (TMUH- BEom5Z6xhybDM99zeLRcwC1LTFmBduv)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}) (2^{A_27a}) (P V0)) (P V1))))$

Definition 4 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27b}). (V0 f V1 g)$

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 6 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 7 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{3}$$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 13 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A.\lambda 27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V2a.($

Definition 19 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 20 We define $c_2Enumeral_2EiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC\ (ap$

Definition 21 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (11)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (12)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (13)$$

Definition 22 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a})$

Definition 23 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 24 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (A_27a)^{A_27b}$

Definition 25 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (A_27a)^{A_27b}$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 26 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ V0n))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (15)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (16)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal} \quad (17)$$

Definition 27 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ V0a))$

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal} \quad (18)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
(19)

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}$$
(20)

Definition 28 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 29 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Definition 30 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 31 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT2))$

Definition 32 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})$$
(21)

Definition 33 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 34 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})$$
(22)

Definition 35 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_neg)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})$$
(23)

Definition 36 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 37 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{(ty_2Enum_2Enum)}$$
(24)

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
(25)

Definition 38 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$.

Definition 39 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$.

Definition 40 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap (ap (ap (c_2Ebool_2ECONF$

Definition 41 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 42 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (26)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric A_27a)^{(ty_2Erealx_2Ereal)^{(ty_2Epair_2Eprod A_27a A_27a)}})) \quad (27)$$

Definition 43 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealx_2Ereal) (ap (c_2E$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}})) \quad (28)$$

Definition 44 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealx_2Ereal)^{(ty_2Epair_2Eprod A_27a A_27a)}})) \quad (29)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (30)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (31)$$

Definition 45 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})_{A_27a})_{(A_27a)^{A_27b}})) \quad (32)$$

Definition 46 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 47 We define $c_2Earithmetic_2EMAX$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2E$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealx_2Ereal^{(ty_2Erealx_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2E)})(33)$$

Definition 48 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 49 We define $c_2Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).(ap (c_2E$

Definition 50 We define $c_2Eseq_2Esummable$ to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).(ap (c_2E$

Definition 51 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})(34)$$

Definition 52 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 53 We define $c_2Epred_set_2Ecount$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Epred_set_2EG$

Definition 54 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 55 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{$

Definition 56 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 57 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m))(35)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge (((ap (ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)))) \wedge (((ap (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC\ V1n)) = (ap\ c_2Enum_2ESUC\ (ap (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))))))))(36)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \quad (38)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) V1m)))))) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)))))) \quad (40)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (41)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((\neg (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((\neg (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = \\
& c_2Enum_2E0) \wedge (V1n = c_2Enum_2E0))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V0m)))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Earithmetic_2E_3E_3D V0n) V1m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Enum_2ESUC V1n)) V0m))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n)))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) c_2Enum_2E0)) \Leftrightarrow (V0n = c_2Enum_2E0))) \wedge (\forall V1m \in ty_2Enum_2Enum. \\
& (\forall V2n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) (ap c_2Enum_2ESUC V2n))) \Leftrightarrow ((V1m = (ap c_2Enum_2ESUC V2n)) \vee \\
& (p (ap (ap c_2Earithmetic_2E_3C_3D V1m) V2n))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\text{True} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{57}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg (p V0t)))) \tag{58}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (59)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A.27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (60)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (61)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (62)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (63)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (64)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (66)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (67)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (68)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (70)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (71)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x))))) \quad (72)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\neg(\exists V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x))))) \quad (73)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (ap\ V1Q\ V4x))))))) \quad (74)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a. (p\ (ap\ V1Q\ V4x))))))) \quad (75)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. (((\exists V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \vee (p\ V1Q)) \Leftrightarrow (\exists V3x \in A_27a. ((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q))))) \quad (76)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p\ V0P) \vee (\exists V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \quad (77)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ & 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A.27a. (p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ & 2^{A.27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ & V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x)))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ & 2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ & V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ & 2. ((\forall V2x \in A.27a. ((p (ap V0P V2x)) \Rightarrow (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A.27a. (p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (\\ & (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee \\ & (p V0A)))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(\\ & p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B)))))))) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee \\ & (p V1B)))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (87)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (88)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0b \in 2. (\forall V1f \in (A_27b^{A_27a}). (\forall V2g \in (A_27b^{A_27a}). \\ & (\forall V3x \in A_27a. ((ap (ap (ap (ap (c_2Ebool_2ECOND (A_27b^{A_27a}) \\ & V0b) V1f) V2g) V3x) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V0b) (ap \\ & V1f V3x)) (ap V2g V3x))))))))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\ & (\forall V3y \in A_27a. ((ap V0f (ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V1b) V2x) V3y) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V1b) (ap V0f \\ & V2x)) (ap V0f V3y))))))))) \end{aligned} \quad (90)$$

Assume the following.

$$(\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p (ap (ap (ap (c_2Ebool_2ECOND 2) V0b) V1t1) V2t2)) \Leftrightarrow (((\neg(p V0b)) \vee (p V1t1)) \wedge ((p V0b) \vee (p V2t2))))))) \quad (91)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27 \\ & V5y_27))))))))) \end{aligned} \quad (93)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (94)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))))) \end{aligned} \quad (95)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). \\ & (\forall V2x \in A_27c. ((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (96)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (97)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (98)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmetic_2EZERO = (ap\ c_2Earithmetic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmetic_2EBIT1\ V0n) = c_2Earithmetic_2EZERO) \Leftrightarrow \\
& False) \wedge ((c_2Earithmetic_2EZERO = (ap\ c_2Earithmetic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmetic_2EBIT2\ V0n) = c_2Earithmetic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmetic_2EBIT1\ V0n) = (ap\ c_2Earithmetic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmetic_2EBIT2\ V0n) = (ap\ c_2Earithmetic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmetic_2EBIT1\ V0n) = (ap\ c_2Earithmetic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmetic_2EBIT2\ V0n) = (ap\ c_2Earithmetic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ c_2Earithmetic_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) \Leftrightarrow \neg(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& V0n)\ V1m))))))))) \\
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\exists V1q \in A_27a. \\
& (\exists V2r \in A_27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\
& V1q)\ V2r)))))) \\
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\
& A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \\
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2ESND\ A_27a \\ & \quad A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (106)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\ & \quad A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ & \quad A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))) \end{aligned} \quad (107)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A_27a)))) \end{aligned} \quad (108)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (109)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V0m)\ (ap\ c_2Epred_set_2Ecount \\ & \quad V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)))) \end{aligned} \quad (110)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27b}). (\forall V2x \in \\ & \quad (ty_2Epair_2Eprod\ A_27a\ A_27b). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ & \quad A_27a\ A_27b)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27a\ A_27b)\ \\ & \quad V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ (ap\ (c_2Epair_2EFST \\ & \quad A_27a\ A_27b)\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ (c_2Epair_2ESND \\ & \quad A_27a\ A_27b)\ V2x))\ V1Q)))))) \end{aligned} \quad (111)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\neg (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & \quad V0n)\ V0n)))) \end{aligned} \quad (112)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg (p (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \quad (113)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add V1y) V0x)))) \quad (114)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z)))))) \quad (115)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x)) \quad (116)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (117)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (118)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul V1y) V0x)))) \quad (119)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z)))))) \quad (120)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x) = V0x)) \quad (121)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x)) \quad (122)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (123)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((\neg(V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \Rightarrow ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Erealax_2Einv V0x)) = (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \quad (124)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add V0x) V2z))) \Leftrightarrow (V1y = V2z)))) \quad (125)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) (ap c_2Erealax_2Ereal_neg V1y)))))) \quad (126)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (127)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) (ap (ap c_2Erealax_2Ereal_add V0x) V2z))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z)))))) \quad (128)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (129)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (130)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (131)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z)))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V2z)))))) \quad (132)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (133)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add V0x) V2z)) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)))))) \quad (134)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. ((ap c_2Ereal_2Ereal_of_num V0m) = (ap c_2Ereal_2Ereal_of_num V1n)) \Leftrightarrow (V0m = V1n)))) \quad (135)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))) \quad (136)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((\neg(V0n = c_2Enum_2E0)) \Rightarrow (p (ap (\\
& ap\ c_2Erealax_2Ereal_lt (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& (ap\ c_2Ereal_2Ereal_of_num\ V0n))))))
\end{aligned} \tag{137}$$

Assume the following.

$$\begin{aligned}
& (\forall V0d \in ty_2Erealax_2Ereal. ((p (ap (ap\ c_2Erealax_2Ereal_lt \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) (ap (ap\ c_2Ereal_2E_2F \\
& V0d) (ap\ c_2Ereal_2Ereal_of_num (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))))) \Leftrightarrow (p (\\
& ap (ap\ c_2Erealax_2Ereal_lt (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& V0d))))
\end{aligned} \tag{138}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap\ c_2Erealax_2Ereal_add \\
& (ap (ap\ c_2Ereal_2E_2F\ V0x) (ap\ c_2Ereal_2Ereal_of_num (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))))) (ap (ap \\
& c_2Ereal_2E_2F\ V0x) (ap\ c_2Ereal_2Ereal_of_num (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))))) = V0x))
\end{aligned} \tag{139}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap\ c_2Erealax_2Ereal_lt (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap\ c_2Erealax_2Ereal_lt (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1y)))) \Rightarrow (p (ap (ap\ c_2Erealax_2Ereal_lt (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap\ c_2Ereal_2E_2F\ V0x) V1y))))))
\end{aligned} \tag{140}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_2Eabs (ap (ap\ c_2Ereal_2Ereal_sub\ V0x) V1y)) = (\\
& ap\ c_2Ereal_2Eabs (ap (ap\ c_2Ereal_2Ereal_sub\ V1y) V0x))))))
\end{aligned} \tag{141}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum. (\forall V1f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\
& ((ap (ap\ c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C\ ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V0n) c_2Enum_2E0)) V1f) = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))) \wedge (\forall V2n \in ty_2Enum_2Enum. (\forall V3m \in \\
& ty_2Enum_2Enum. (\forall V4f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\
& ((ap (ap\ c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C\ ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2n) (ap\ c_2Enum_2ESUC\ V3m))) V4f) = (ap (ap\ c_2Erealax_2Ereal_add \\
& (ap (ap\ c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C\ ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2n) V3m)) V4f)) (ap\ V4f (ap (ap\ c_2Earithmetic_2E_2B \\
& V2n) V3m)))))))))
\end{aligned} \tag{142}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V2m \in ty_2Enum_2Enum. \\
& (\forall V3n \in ty_2Enum_2Enum.((\forall V4r \in ty_2Enum_2Enum. \\
& (((p (ap (ap (ap c_2Earithmic_2E_3C_3D V2m) V4r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
& V4r) (ap (ap c_2Earithmic_2E_2B V3n) V2m)))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap V0f V4r)) (ap V1g V4r)))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (\\
& ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) \\
& V2m) V3n)) V0f)) (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C \\
& ty_2Enum_2Enum ty_2Enum_2Enum) V2m) V3n)) V1g))))))))) \\
& \hspace{15em} (143)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1N \in \\
& ty_2Enum_2Enum.((\forall V2n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3E_3D \\
& V2n) V1N)) \Rightarrow ((ap V0f V2n) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \Rightarrow \\
& (\forall V3m \in ty_2Enum_2Enum.(\forall V4n \in ty_2Enum_2Enum.(\\
& (p (ap (ap c_2Earithmic_2E_3E_3D V3m) V1N)) \Rightarrow ((ap (ap c_2Ereal_2Esum \\
& (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V3m) V4n)) \\
& V0f) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))))) \\
& \hspace{15em} (144)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V2m \in ty_2Enum_2Enum. \\
& (\forall V3n \in ty_2Enum_2Enum.((ap (ap c_2Ereal_2Esum (ap (ap (\\
& c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V2m) V3n)) (\lambda V4n \in \\
& ty_2Enum_2Enum.(ap (ap c_2Erealax_2Ereal_add (ap V0f V4n)) (\\
& ap V1g V4n)))))) = (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Ereal_2Esum \\
& (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V2m) V3n)) \\
& V0f)) (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2m) V3n)) V1g))))))))) \\
& \hspace{15em} (145)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V0m) V1n)) (\lambda V2r \in ty_2Enum_2Enum.(ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \\
& \hspace{15em} (146)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V1y)))))) \\
& \hspace{15em} (147)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0y) V1x))))))
\end{aligned} \tag{148}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V2z)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap (ap c_2Erealax_2Ereal_add V0x) V2z))))))
\end{aligned} \tag{149}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y))))))
\end{aligned} \tag{150}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x))))))
\end{aligned} \tag{151}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Erealax_2Ereal_neg V0x)) = V0x))
\end{aligned} \tag{152}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{153}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{154}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
V0m)) (ap c_2Ereal_2Ereal_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0m) V1n))))))
\end{aligned} \tag{155}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (((ap c_2Ereal_2Ereal_of_num V0n) = (ap c_2Ereal_2Ereal_of_num \\
V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num \\
& V0n)) = (ap c_2Ereal_2Ereal_of_num V1m)) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge \\
& (V1m = c_2Enum_2E0))) \wedge (((ap c_2Ereal_2Ereal_of_num V0n) = \\
& (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num V1m))) \Leftrightarrow \\
& ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Ereal_2Ereal_of_num V0n)) = (ap c_2Erealax_2Ereal_neg \\
& (ap c_2Ereal_2Ereal_of_num V1m))) \Leftrightarrow (V0n = V1m))))))
\end{aligned} \tag{156}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{157}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{158}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{159}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{160}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{161}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{162}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{163}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{164}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{165}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{166}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{167}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{168}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{169}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{170}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{171}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V1x0 \in \\
& ty_2Erealax_2Ereal. ((p \ (ap \ (ap \ c_2Eseq_2E_2D_2D_3E \ V0x) \ V1x0)) \Leftrightarrow \\
& (\forall V2e \in ty_2Erealax_2Ereal. ((p \ (ap \ (ap \ c_2Erealax_2Ereal_It \\
& (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)) \ V2e)) \Rightarrow (\exists V3N \in \\
& ty_2Enum_2Enum. (\forall V4n \in ty_2Enum_2Enum. ((p \ (ap \ (ap \ c_2Earithmic_2E_3E_3D \\
& V4n) \ V3N)) \Rightarrow (p \ (ap \ (ap \ c_2Erealax_2Ereal_It \ (ap \ c_2Ereal_2Eabs \\
& (ap \ (ap \ c_2Ereal_2Ereal_sub \ (ap \ V0x \ V4n)) \ V1x0))) \ V2e)))))))))
\end{aligned} \tag{172}$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).((p (ap c_2Eseq_2Esummable V0f)) \Rightarrow (p (ap (ap c_2Eseq_2Esums V0f) (ap c_2Eseq_2Esuminf V0f)))))) \quad (173)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1n \in \\ & ty_2Enum_2Enum.(((p (ap c_2Eseq_2Esummable V0f)) \wedge (\forall V2m \in \\ & ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2m)) \Rightarrow \\ & (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0)) (ap V0f V2m)))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\ & (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\ & ty_2Enum_2Enum) c_2Enum_2E0) V1n)) V0f)) (ap c_2Eseq_2Esuminf \\ & V0f)))))) \end{aligned} \quad (174)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1l \in \\ & ty_2Erealax_2Ereal.(((\forall V2n \in ty_2Enum_2Enum.(p (ap (ap \\ & c_2Ereal_2Ereal_lte (ap V0f V2n)) (ap V0f (ap c_2Enum_2ESUC V2n)))))) \wedge \\ & ((\forall V3n \in ty_2Enum_2Enum.(p (ap (ap c_2Ereal_2Ereal_lte \\ & (ap V0f V3n)) V1l))) \wedge (\forall V4e \in ty_2Erealax_2Ereal.((p (ap \\ & (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\ & V4e)) \Rightarrow (\exists V5n \in ty_2Enum_2Enum.(p (ap (ap c_2Erealax_2Ereal_lt \\ & V1l) (ap (ap c_2Erealax_2Ereal_add (ap V0f V5n)) V4e)))))) \Rightarrow \\ & (p (ap (ap c_2Eseq_2E_2D_2D_3E V0f) V1l)))))) \end{aligned} \quad (175)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \forall V2k \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V2k) (ap (ap c_2Earithmetic_2EMAX V0m) V1n))) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V2k) V0m)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D V2k) V1n)))))) \end{aligned} \quad (176)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(((p (ap c_2Eseq_2Esummable \\ & V0f)) \wedge (\forall V1n \in ty_2Enum_2Enum.(p (ap (ap c_2Ereal_2Ereal_lte \\ & (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap V0f V1n)))))) \Rightarrow \\ & (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0)) (ap c_2Eseq_2Esuminf V0f)))) \end{aligned} \quad (177)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1x \in \\ & ty_2Erealax_2Ereal.(((p (ap (ap c_2Eseq_2Esums V0f) V1x)) \Leftrightarrow ((p \\ & (ap c_2Eseq_2Esummable V0f)) \wedge ((ap c_2Eseq_2Esuminf V0f) = V1x)))))) \end{aligned} \quad (178)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\
& \quad \forall V2x \in ty_2Erealx_2Ereal. ((ap (ap c_2Ereal_2Esum (ap (\\
& \quad \quad ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) c_2Enum_2E0) \\
& \quad V0n)) (\lambda V3m \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Erealx_2Ereal) \\
& \quad (ap (ap (c_2Emin_2E_3D ty_2Enum_2Enum) V3m) V1k)) V2x) (ap c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0)))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Erealx_2Ereal) \\
& \quad (ap (ap c_2Eprim_rec_2E_3C V1k) V0n)) V2x) (ap c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0))))))
\end{aligned} \tag{179}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}). (\forall V1g \in \\
& \quad (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}). (\forall V2m \in ty_2Enum_2Enum. \\
& \quad \quad (\forall V3n \in ty_2Enum_2Enum. (((\forall V4r \in ty_2Enum_2Enum. \\
& \quad \quad \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D V2m) V4r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad \quad \quad V4r) (ap (ap c_2Earithmetic_2E_2B V3n) V2m)))) \Rightarrow (p (ap (ap c_2Erealx_2Ereal_lt \\
& \quad \quad \quad (ap V0f V4r)) (ap V1g V4r)))))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& \quad \quad \quad V3n))) \Rightarrow (p (ap (ap c_2Erealx_2Ereal_lt (ap (ap c_2Ereal_2Esum \\
& \quad \quad \quad (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V2m) V3n)) \\
& \quad \quad \quad V0f)) (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Enum_2Enum) V2m) V3n)) V1g))))))
\end{aligned} \tag{180}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1r \in ty_2Erealx_2Ereal. \\
& \quad ((ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& \quad \quad ty_2Enum_2Enum) c_2Enum_2E0) V0n)) (ap (c_2Ecombin_2EK ty_2Erealx_2Ereal \\
& \quad \quad ty_2Enum_2Enum) V1r)) = (ap (ap c_2Erealx_2Ereal_mul (ap c_2Ereal_2Ereal_of_num \\
& \quad \quad \quad V0n)) V1r))))
\end{aligned} \tag{181}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Enum_2Enum. (\forall V1b \in ty_2Enum_2Enum. (\\
& \quad (p (ap (ap c_2Eprim_rec_2E_3C V0a) (ap c_2Enum_2ESUC V1b))) \Leftrightarrow (\\
& \quad \quad (p (ap (ap c_2Eprim_rec_2E_3C V0a) V1b)) \vee (V0a = V1b))))
\end{aligned} \tag{182}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. ((ap (c_2Ecombin_2EK A_27a A_27b) V0x) = (\lambda V1z \in \\
& \quad \quad A_27b. V0x)))
\end{aligned} \tag{183}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Enum_2Enum}). \\
& \quad (\forall V1N \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ \\
& \quad \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) \\
& \quad \quad V0f)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))\ (ap\ (ap\ (c_2Epred_set_2ECROSS \\
& \quad \quad ty_2Enum_2Enum\ ty_2Enum_2Enum)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)) \\
& \quad \quad (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)))) \Rightarrow (\exists V2k \in ty_2Enum_2Enum. \\
& \quad (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad \quad ty_2Enum_2Enum))\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ ty_2Enum_2Enum \\
& \quad \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ V0f)\ (ap\ c_2Epred_set_2Ecount \\
& \quad \quad V1N)))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ ty_2Enum_2Enum\ ty_2Enum_2Enum) \\
& \quad \quad (ap\ c_2Epred_set_2Ecount\ V2k))\ (ap\ c_2Epred_set_2Ecount\ V2k))))))))) \\
& \hspace{15em} (184)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Enum_2Enum}). \\
& \quad (\forall V1k \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ \\
& \quad \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) \\
& \quad \quad V0f)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))\ (ap\ (ap\ (c_2Epred_set_2ECROSS \\
& \quad \quad ty_2Enum_2Enum\ ty_2Enum_2Enum)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)) \\
& \quad \quad (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)))) \Rightarrow (\exists V2N \in ty_2Enum_2Enum. \\
& \quad (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad \quad ty_2Enum_2Enum))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ ty_2Enum_2Enum \\
& \quad \quad ty_2Enum_2Enum)\ (ap\ c_2Epred_set_2Ecount\ V1k))\ (ap\ c_2Epred_set_2Ecount \\
& \quad \quad V1k)))\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad \quad ty_2Enum_2Enum\ ty_2Enum_2Enum))\ V0f)\ (ap\ c_2Epred_set_2Ecount \\
& \quad \quad V2N))))))))) \\
& \hspace{15em} (185)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\
& \quad (\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V2h \in \\
& \quad ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Enum_2Enum}). \\
& \quad \quad ((\forall V3m \in ty_2Enum_2Enum. (\forall V4n \in ty_2Enum_2Enum. \\
& \quad \quad \quad (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad \quad c_2Enum_2E0))\ (ap\ (ap\ V0f\ V3m)\ V4n)))))) \wedge ((\forall V5n \in ty_2Enum_2Enum. \\
& \quad (p\ (ap\ (ap\ c_2Eseq_2Esums\ (ap\ V0f\ V5n))\ (ap\ V1g\ V5n)))) \wedge ((p\ (ap\ c_2Eseq_2Esummable \\
& \quad \quad V1g)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad \quad \quad ty_2Enum_2Enum\ ty_2Enum_2Enum))\ V2h)\ (c_2Epred_set_2EUNIV \\
& \quad \quad \quad ty_2Enum_2Enum))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Enum_2Enum)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))\ (c_2Epred_set_2EUNIV \\
& \quad \quad \quad ty_2Enum_2Enum)))))) \Rightarrow (p\ (ap\ (ap\ c_2Eseq_2Esums\ (ap\ (ap\ (c_2Ecombin_2Eo \\
& \quad \quad \quad ty_2Enum_2Enum\ ty_2Erealax_2Ereal\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Enum_2Enum))\ (ap\ (c_2Epair_2EUNCURRY\ ty_2Enum_2Enum\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Erealax_2Ereal)\ V0f))\ V2h))\ (ap\ c_2Eseq_2Esuminf\ V1g)))))))))
\end{aligned}$$