thm_2Eseq_2EX__HALF__HALF (TMQpaSvNr-RgCbAKsS9AZJ8UZ6JXB95uRZnF)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2EF

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$ (

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (7)$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(8)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

Definition 18 *We define* c_2Enumeral_2EiiSUC *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2ESUC\ (ap$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

Definition 19 *We define* c_2Enumeral_2EiDUB *to be* $\lambda V0x \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2$

Definition 20 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \qquad (11)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$

$$(12)$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \qquad (13)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}$$

$$(14)$$

Definition 21 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 (t

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh}$$

$$(15)$$

Definition 22 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal$$
$$ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})$$

$$(16)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh}$$

$$(17)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr}}}$$

$$(18)$$

Definition 23 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 24 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal.

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal$$
$$ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh}$$

$$(19)$$

Definition 25 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealа$

Definition 26 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 27 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{20}$$

Definition 28 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal$$
$$ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{21}$$

Definition 29 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal$$
$$ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh}} \tag{22}$$

Definition 30 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Definition 31 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Definition 32 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 33 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 34 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{23}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n))) \tag{24}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V1n)\ V0m))))) \tag{25}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_2B}\ V0m)\ V1n) = \textsf{c\_2Enum\_2E0}) \Leftrightarrow ((V0m =$$
$$\textsf{c\_2Enum\_2E0}) \wedge (V1n = \textsf{c\_2Enum\_2E0})))))$$

$$(26)$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ \textsf{c\_2Enum\_2E0})) \Leftrightarrow (V0n = \textsf{c\_2Enum\_2E0}))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.$$
$$(\forall V2n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V2n))) \Leftrightarrow ((V1m = (ap\ \textsf{c\_2Enum\_2ESUC}\ V2n)) \vee$$
$$(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V2n)))))))))$$

$$(27)$$

Assume the following.
$$True \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge$$
$$(((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))))$$

$$(29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow$$
$$True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (($$
$$(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t))))))))$$

$$(30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge$$
$$((\neg False) \Leftrightarrow True)))$$

$$(31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow$$
$$True))$$

$$(32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in$$
$$A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))$$

$$(33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg ($$
$$p\ V0t)))))))$$

$$(34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B)))))))) \tag{35}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$\text{c\_2Enum\_2E0})\ V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \text{c\_2Enum\_2E0}) = V1n)) \wedge ((\forall V2n \in$$
$$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V3m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Enumeral\_2EiZ}\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \text{c\_2Enum\_2E0})\ V4n) = \text{c\_2Enum\_2E0})) \wedge$$
$$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V5n)\ \text{c\_2Enum\_2E0}) = \text{c\_2Enum\_2E0})) \wedge ((\forall V6n \in ty\_2Enum\_2Enum.$$
$$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($$
$$ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V7m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V6n)\ V7m))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$\text{c\_2Enum\_2E0})\ V8n) = \text{c\_2Enum\_2E0})) \wedge ((\forall V9n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \text{c\_2Enum\_2E0}) = V9n)) \wedge ((\forall V10n \in$$
$$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V11m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V10n)\ V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$\text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}$$
$$V12n))) = \text{c\_2Enum\_2E0})) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2EEXP\ \text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \text{c\_2Earithmetic\_2EBIT2}\ V13n))) = \text{c\_2Enum\_2E0})) \wedge ((\forall V14n \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \text{c\_2Enum\_2E0}) =$$
$$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}\ \text{c\_2Earithmetic\_2EZERO})))) \wedge$$
$$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V15n))$$
$$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))) \wedge (((ap\ \text{c\_2Enum\_2ESUC}$$
$$\text{c\_2Enum\_2E0}) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}$$
$$\text{c\_2Earithmetic\_2EZERO}))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.($$
$$(ap\ \text{c\_2Enum\_2ESUC}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \text{c\_2Enum\_2ESUC}\ V17n)))) \wedge (((ap\ \text{c\_2Eprim\_\_rec\_2EPRE}\ \text{c\_2Enum\_2E0}) =$$
$$\text{c\_2Enum\_2E0}) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \text{c\_2Eprim\_\_rec\_2EPRE}$$
$$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \text{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum.$$
$$(((ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \text{c\_2Enum\_2E0}) \Leftrightarrow (V19n = \text{c\_2Earithmetic\_2EZERO}))) \wedge$$
$$((\forall V20n \in ty\_2Enum\_2Enum.((\text{c\_2Enum\_2E0} = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V20n)) \Leftrightarrow (V20n = \text{c\_2Earithmetic\_2EZERO}))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum.$$
$$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V21n) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V22m)) \Leftrightarrow (V21n = V22m))))) \wedge$$
$$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V23n)\ \text{c\_2Enum\_2E0})) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V24n))) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Earithmetic\_2EZERO})$$
$$V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V25n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V26m))) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V25n)\ V26m))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}$$
$$\text{c\_2Enum\_2E0})\ V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V28n))\ \text{c\_2Enum\_2E0})) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Earithmetic\_2EZERO})$$
$$V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V29n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V30m))) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V30m)\ V29n))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\text{c\_2Enum\_2E0})\ V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum.$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})\\
V0n)) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (\\
ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (\\
ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (\\
ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge\\
(((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})\\
V0n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap\\
\mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
(ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge\\
(((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})\\
V0n)) = (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (\\
ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (\\
ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) =\\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) =\\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0n)\ V1m)))) \wedge ((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) =\\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0n)\ V1m)))))))))))))))))))))))))$$

(37)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$

$$((p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}~\textsf{c\_2Earithmetic\_2EZERO})~(ap~\textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n)))\Leftrightarrow True)\wedge(((p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}~\textsf{c\_2Earithmetic\_2EZERO})$$
$$(ap~\textsf{c\_2Earithmetic\_2EBIT2}~V0n)))\Leftrightarrow True)\wedge(((p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n)~\textsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap~\textsf{c\_2Earithmetic\_2EBIT1}~V0n))~(ap~\textsf{c\_2Earithmetic\_2EBIT1}~V1m)))\Leftrightarrow$$
$$(p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}~V0n)~V1m)))\wedge(((p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap~\textsf{c\_2Earithmetic\_2EBIT2}~V0n))~(ap~\textsf{c\_2Earithmetic\_2EBIT2}~V1m)))\Leftrightarrow$$
$$(p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}~V0n)~V1m)))\wedge(((p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap~\textsf{c\_2Earithmetic\_2EBIT1}~V0n))~(ap~\textsf{c\_2Earithmetic\_2EBIT2}~V1m)))\Leftrightarrow$$
$$(\neg(p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}~V1m)~V0n))))\wedge((p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap~\textsf{c\_2Earithmetic\_2EBIT2}~V0n))~(ap~\textsf{c\_2Earithmetic\_2EBIT1}~V1m)))\Leftrightarrow$$
$$(p~(ap~(ap~\textsf{c\_2Eprim\_\_rec\_2E\_3C}~V0n)~V1m)))))))))))$$

(38)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(((ap~\textsf{c\_2Enumeral\_2EiDUB}~(ap~\textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n)) = (ap~\textsf{c\_2Earithmetic\_2EBIT2}~(ap~\textsf{c\_2Enumeral\_2EiDUB}~V0n)))\wedge$$
$$(((ap~\textsf{c\_2Enumeral\_2EiDUB}~(ap~\textsf{c\_2Earithmetic\_2EBIT2}~V0n)) = (ap$$
$$\textsf{c\_2Earithmetic\_2EBIT2}~(ap~\textsf{c\_2Earithmetic\_2EBIT1}~V0n)))\wedge((ap$$
$$\textsf{c\_2Enumeral\_2EiDUB}~\textsf{c\_2Earithmetic\_2EZERO}) = \textsf{c\_2Earithmetic\_2EZERO}))))$$

(39)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$

$$((ap~(ap~c\_2Earithmetic\_2E\_2A~\textsf{c\_2Earithmetic\_2EZERO})~V0n) = \textsf{c\_2Earithmetic\_2EZERO})\wedge$$
$$(((ap~(ap~c\_2Earithmetic\_2E\_2A~V0n)~\textsf{c\_2Earithmetic\_2EZERO}) =$$
$$\textsf{c\_2Earithmetic\_2EZERO})\wedge(((ap~(ap~c\_2Earithmetic\_2E\_2A~(ap~\textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))~V1m) = (ap~\textsf{c\_2Enumeral\_2EiZ}~(ap~(ap~c\_2Earithmetic\_2E\_2B$$
$$(ap~\textsf{c\_2Enumeral\_2EiDUB}~(ap~(ap~c\_2Earithmetic\_2E\_2A~V0n)~V1m)))$$
$$V1m)))\wedge((ap~(ap~c\_2Earithmetic\_2E\_2A~(ap~\textsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))~V1m) = (ap~\textsf{c\_2Enumeral\_2EiDUB}~(ap~\textsf{c\_2Enumeral\_2EiZ}~(ap~(ap$$
$$c\_2Earithmetic\_2E\_2B~(ap~(ap~c\_2Earithmetic\_2E\_2A~V0n)~V1m))$$
$$V1m)))))))))$$

(40)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((ap~(ap~\textsf{c\_2Erealax\_2Ereal\_\_add}~V0x)~V1y) = (ap~(ap~\textsf{c\_2Erealax\_2Ereal\_\_add}$$
$$V1y)~V0x))))$$

(41)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_add}$$
$$V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_add}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_add}$$
$$(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_add}\ V0x)\ V1y))\ V2z)))))$$

(42)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_add}$$
$$(ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = V0x))$$

(43)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_add}$$
$$(ap\ \mathsf{c\_2Erealax\_2Ereal\_neg}\ V0x))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})))$$

(44)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_lt}$$
$$V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_lt}\ V1y)\ V2z)))\Rightarrow(p\ (ap$$
$$(ap\ \mathsf{c\_2Erealax\_2Ereal\_lt}\ V0x)\ V2z))))))$$

(45)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_mul}$$
$$V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_mul}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_mul}$$
$$(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_mul}\ V0x)\ V1y))\ V2z)))))$$

(46)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_mul}$$
$$(ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x) = V0x))$$

(47)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ V0x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ V1y)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_mul}\ V0x)\ V1y))))))$$

(48)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_add}$$
$$V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = V0x))$$

(49)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}$$
$$V0x)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})))$$

(50)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)$$
$$V1y)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}$$
$$V0x))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y)))))$$

(51)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}$$
$$(ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})))$$

(52)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}$$
$$V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})))$$

(53)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}$$
$$(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}$$
$$V0x)\ V2z))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1y)\ V2z))))))$$

(54)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V0x)\ V0x)))$$

(55)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}$$
$$V0x)\ V1y)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V2z))) \Rightarrow (p\ (ap\ ($$
$$ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V2z))))))$$

(56)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V0x)\ V1y)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1y)\ V2z))) \Rightarrow (p\ (ap$$
$$(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V2z))))))$$

(57)

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Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.(((p \ (ap \ (ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V0x) \ V1y)) \wedge (p \ (ap \ (ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte} \ V1y) \ V2z))) \Rightarrow (p \ (ap \ ($$
$$ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte} \ V0x) \ V2z))))))$$

(58)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((((p \ (ap \ (ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte} \ V0x) \ V1y)) \wedge (p \ (ap \ (ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V1y) \ V0x))) \Leftrightarrow (V0x = V1y))))$$

(59)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((((p \ (ap \ (ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte} \ (ap \ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) \ V0x)) \wedge (p \ (ap \ (ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte} \ (ap \ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) \ V1y))) \Rightarrow (p \ (ap \ (ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte} \ (ap \ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) \ (ap \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \ V0x) \ V1y))))))$$

(60)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(ap \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_add} \ (ap \ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$V0m)) \ (ap \ c\_2Ereal\_2Ereal\_\_of\_\_num \ V1n)) = (ap \ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$(ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0m) \ V1n)))))$$

(61)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((ap \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \ V0x) \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_neg}$$
$$V1y)) = (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_neg} \ (ap \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_mul}$$
$$V0x) \ V1y)))))$$

(62)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((ap \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_neg} \ V0x))$$
$$V1y) = (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_neg} \ (ap \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_mul}$$
$$V0x) \ V1y)))))$$

(63)

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal.$$
$$((p \ (ap \ (ap \ \mathsf{c\_2Erealax\_2Ereal\_\_lt} \ V1x) \ V0y)) \Leftrightarrow (\neg(p \ (ap \ (ap \ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V0y) \ V1x))))))$$

(64)

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Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V1y)\ V2z))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}$$
$$V0x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V2z)))))))$$

(65)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))$$
$$V1y))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_of\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))))))$$

(66)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))$$
$$(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V1y)\ V0x)))))$$

(67)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}$$
$$(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = V0x))$$

(68)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}$$
$$V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}$$
$$V0x)\ V1y))\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ \mathsf{c\_2Enum\_2E0}))))))$$

(69)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}$$
$$V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}$$
$$(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}$$
$$V0x)\ V2z))))))$$

(70)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}$$
$$(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}$$
$$(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}$$
$$V1y)\ V2z))))))$$

(71)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\\ V0m))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\ V0m)\ V1n)))))$$

(72)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\\ V0m))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1n)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_2A}\ V0m)\ V1n)))))$$

(73)

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))$$

(74)

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False)))$$

(75)

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow\\ (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))$$

(76)

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow\\ ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))$$

(77)

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False)))$$

(78)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(\\ p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee\\ ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))))$$

(79)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee\\ (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p)))))))))))$$

(80)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}(\\ (p\ V1q){\vee}(p\ V2r)))){\Leftrightarrow}(((p\ V0p){\vee}({\neg}(p\ V1q)))){\wedge}(((p\ V0p){\vee}({\neg}(p\ V2r)))){\wedge}\\ ((p\ V1q){\vee}((p\ V2r){\vee}({\neg}(p\ V0p))))))))))$$

$$(81)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}(\\ (p\ V1q){\Rightarrow}(p\ V2r)))){\Leftrightarrow}(((p\ V0p){\vee}(p\ V1q)){\wedge}(((p\ V0p){\vee}({\neg}(p\ V2r)))){\wedge}((\\ {\neg}(p\ V1q)){\vee}((p\ V2r){\vee}({\neg}(p\ V0p))))))))))$$

$$(82)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p){\Leftrightarrow}({\neg}(p\ V1q))){\Leftrightarrow}(((p\ V0p){\vee}\\ (p\ V1q)){\wedge}(({\neg}(p\ V1q)){\vee}({\neg}(p\ V0p)))))))$$

$$(83)$$

Assume the following.

$$((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))=\\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\\ ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))$$

$$(84)$$

Theorem 1

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\\ ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x))\ (\\ ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\\ ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x))=\\ V0x))$$