

thm_2Eset_relation_2Eacyclic_bigunion (TMVo- jptHXjAqZf8GoTnMac8wVYPPDocYeQj)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$.

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 5 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define `c_2Epred_set_2EEMPTY` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. \text{c_2Ebool_2E_2F})$.

Definition 7 We define `c_2Ebool_2E_2IN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

Definition 8 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 \ A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EABS_prod } A. 27a \ A. 27b \in ((\text{ty_2Epair_2Eprod } A. 27a \ A. 27b))^{((2^{A-27b})^{A-27a})} \tag{2}$$

Definition 10 We define `c_2Epair_2E_2C` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0x \in A. 27a. \lambda V1y \in A. 27b. (\text{ap } (\text{c_2Epair_2E_2C } A. 27a \ A. 27b))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (3)$$

Definition 11 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(p\ V1t2 \Rightarrow (p\ V0t1 \Rightarrow (p\ V1t2))))))$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(p\ V1t2 \Rightarrow (p\ V0t1 \Rightarrow (p\ V1t2)))))))$

Definition 13 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(p\ V1t2 \Rightarrow (p\ V0t1 \Rightarrow (p\ V1t2))))))$

Definition 14 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(p\ V1t2 \Rightarrow (p\ V0t1 \Rightarrow (p\ V1t2))))))$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ 40)\ (\lambda V1t \in 2.(p\ V1t2 \Rightarrow (p\ V0t1 \Rightarrow (p\ V1t2)))))))$

Definition 16 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set_2EUNION)\ (\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(p\ V1t2 \Rightarrow (p\ V0t1 \Rightarrow (p\ V1t2)))))))$

Definition 17 We define $c_2Eset_relation_2Erange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).$

Definition 18 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).$

Definition 19 We define $c_2Eset_relation_2Etc$ to be $\lambda A_27a : \iota.(\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})).$

Definition 20 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ (c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2)\ (\lambda V1t \in 2.(p\ V1t2 \Rightarrow (p\ V0t1 \Rightarrow (p\ V1t2)))))))$

Definition 21 We define $c_2Eset_relation_2Eacyclic$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}).$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))))) \end{aligned} \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (9)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (10)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p(ap V0P V2x)) \wedge (p(ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a.(p(ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p(ap V1Q V4x))))))) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((p V0P) \wedge (\forall V2x \in A.27a.(p(ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p(ap V1Q V3x)))))) \quad (15)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A.27a}).((\forall V2x \in A.27a.((p(ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p(ap V1P V3x))) \vee (p V0Q)))))) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \vee (p(ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p(ap V1Q V3x)))))) \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & 2. ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A_27a. (p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ & p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ & (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge \\ & (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). (\forall V2u \in (2^{A_27a}). (((p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT \\
& A_27a)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))\ V2u)) \Leftrightarrow \\
& ((p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT\ A_27a)\ V0s)\ V2u)) \wedge (p\ (ap \\
& (ap\ (c_2Epred_set_2EDISJOINT\ A_27a)\ V1t)\ V2u)))) \wedge ((p\ (ap\ (ap \\
& (c_2Epred_set_2EDISJOINT\ A_27a)\ V2u)\ (ap\ (ap\ (c_2Epred_set_2EUNION \\
& A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT\ A_27a) \\
& V0s)\ V2u)) \wedge (p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT\ A_27a)\ V1t)\ V2u))))))))) \\
& \tag{27}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1sos \in \\
& (2^{(2^{A_27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGUNION \\
& A_27a)\ V1sos))) \Leftrightarrow (\exists V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2s)\ V1sos))))))))) \\
& \tag{28}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \tag{31}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \tag{32}
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& (p\ V2r)) \vee (\neg(p\ V1q)))))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))) \\
& \tag{34}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee \neg(p \vee V2r)))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee \neg(p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow \neg(p \vee V1q)) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge (\neg(p \vee V1q) \vee \neg(p \vee V0p)))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p)))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow \neg(p \vee V1q)))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V0p)))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V1q)))) \\
& \hspace{15em} (42)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \\
& \hspace{15em} (43)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}), \\
& \quad ((\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\
& \quad A_27a)\ V1x)\ V2y))\ V0r)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V1x)\ V2y))\ (ap \\
& \quad (c_2Eset_relation_2Etc\ A_27a)\ V0r)))))) \wedge (\forall V3x \in A_27a. \\
& \quad (\forall V4y \in A_27a. ((\exists V5z \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\
& \quad A_27a)\ V3x)\ V5z))\ (ap\ (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \wedge (\\
& \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A_27a\ A_27a)\ V5z)\ V4y))\ (ap\ (c_2Eset_relation_2Etc \\
& \quad A_27a)\ V0r)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a \\
& \quad A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V3x)\ V4y))\ (ap\ (c_2Eset_relation_2Etc \\
& \quad A_27a)\ V0r)))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}), \\
& \quad (\forall V1tc_27 \in ((2^{A_27a})^{A_27a}). ((\forall V2x \in A_27a. (\forall V3y \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V2x)\ V3y))\ V0r)) \Rightarrow (p\ (ap\ (ap \\
& \quad V1tc_27\ V2x)\ V3y)))))) \wedge (\forall V4x \in A_27a. (\forall V5y \in A_27a. \\
& \quad ((\exists V6z \in A_27a. ((p\ (ap\ (ap\ V1tc_27\ V4x)\ V6z)) \wedge (p\ (ap\ (ap\ V1tc_27 \\
& \quad V6z)\ V5y)))))) \Rightarrow (p\ (ap\ (ap\ V1tc_27\ V4x)\ V5y)))))) \Rightarrow (\forall V7x \in A_27a. \\
& \quad (\forall V8y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V7x)\ V8y))\ (ap \\
& \quad (c_2Eset_relation_2Etc\ A_27a)\ V0r)) \Rightarrow (p\ (ap\ (ap\ V1tc_27\ V7x) \\
& \quad V8y)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}), \\
& \quad (\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\
& \quad A_27a)\ V1x)\ V2y))\ (ap\ (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \Rightarrow (\\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ (ap\ (c_2Eset_relation_2Edomain \\
& \quad A_27a\ A_27a)\ V0r))) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2y)\ (ap\ (c_2Eset_relation_2Erange \\
& \quad A_27a\ A_27a)\ V0r)))))))))
\end{aligned} \tag{46}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0rs \in (2^{(2^{(ty_2Epair_2Eprod } A_{27a} A_{27a}))}). \\
& \quad (((\forall V1r \in (2^{(ty_2Epair_2Eprod } A_{27a} A_{27a)}), (\forall V2r_{27} \in \\
& \quad (2^{(ty_2Epair_2Eprod } A_{27a} A_{27a)}), ((p (ap (ap (c_2Ebool_2EIN \\
& (2^{(ty_2Epair_2Eprod } A_{27a} A_{27a)}) V1r) V0rs)) \wedge ((p (ap (ap (c_2Ebool_2EIN \\
& (2^{(ty_2Epair_2Eprod } A_{27a} A_{27a)}) V2r_{27}) V0rs)) \wedge (\neg(V1r = V2r_{27})))) \Rightarrow \\
& (p (ap (ap (c_2Epred_set_2EDISJOINT } A_{27a}) (ap (ap (c_2Epred_set_2EUNION \\
& A_{27a}) (ap (c_2Eset_relation_2Edomain } A_{27a} A_{27a}) V1r)) (ap \\
& (c_2Eset_relation_2Erange } A_{27a} A_{27a}) V1r))) (ap (ap (c_2Epred_set_2EUNION \\
& A_{27a}) (ap (c_2Eset_relation_2Edomain } A_{27a} A_{27a}) V2r_{27})) \\
& (ap (c_2Eset_relation_2Erange } A_{27a} A_{27a}) V2r_{27})))))) \wedge (\\
& \quad \forall V3r \in (2^{(ty_2Epair_2Eprod } A_{27a} A_{27a)}), ((p (ap (ap (c_2Ebool_2EIN \\
& (2^{(ty_2Epair_2Eprod } A_{27a} A_{27a)}) V3r) V0rs)) \Rightarrow (p (ap (c_2Eset_relation_2Eacyclic \\
& A_{27a}) V3r)))) \Rightarrow (p (ap (c_2Eset_relation_2Eacyclic } A_{27a}) (\\
& \quad ap (c_2Epred_set_2EBIGUNION } (ty_2Epair_2Eprod } A_{27a} A_{27a}) \\
& \quad V0rs))))))
\end{aligned}$$