

# thm\_2Eset\_relation\_2Eacyclic\_reln\_to\_rel\_conv (TMdUCB- JWqpU8xPcVsxMH5dPTtdrNwVZSHjJ)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}}))$

**Definition 4** We define `c_2Ebool_2E_2F` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

**Definition 7** We define `c_2Erelation_2Eirreflexive` to be  $\lambda A_{27a} : \iota.\lambda V0R \in ((2^{A_{27a}})^{A_{27a}}).(ap (c_2Ebool_2E_7E$

**Definition 8** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow c\_2Epair\_2EABS\_prod A_{27a} A_{27b} \in ((ty\_2Epair\_2Eprod A_{27a} A_{27b})^{(2^{A_{27b}})^{A_{27a}}}) \quad (2)$$

**Definition 9** We define `c_2Epair_2E_2C` to be  $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda V0x \in A_{27a}.\lambda V1y \in A_{27b}.(ap (c_2Ebool_2E_7E$

**Definition 10** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 12** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_Eset\_relation\_2Etc$  to be  $\lambda A\_27a : \iota. (\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). ($

**Definition 14** We define  $c\_Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x))$

**Definition 15** We define  $c\_Eset\_relation\_2Eacyclic$  to be  $\lambda A\_27a : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (3)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (4)$$

**Definition 16** We define  $c\_Eset\_relation\_2Ereln\_to\_rel$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2E$

**Definition 17** We define  $c\_Erelation\_2ETC$  to be  $\lambda A\_27a : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). \lambda V1a \in A\_27a. \lambda V2b$

**Definition 18** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c^{A\_27a}$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (5)$$

**Definition 19** We define  $c\_Eset\_relation\_2Erel\_to\_reln$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27b})^{A\_27a}$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap\ (c\_2Epair\_2EFST\ A\_27a \\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b) \\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0xy \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b). (\forall V1R \in ( \\ 2^{A\_27b})^{A\_27a}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0xy) \\ (ap\ (c\_2Eset\_relation\_2Erel\_to\_reln\ A\_27a\ A\_27b)\ V1R))) \Leftrightarrow (p\ (ap\ (ap\ V1R\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0xy)) \\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V0xy)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). (\forall V1x \in \\ A\_27a. (\forall V2y \in A\_27b. ((p\ (ap\ (ap\ (ap\ (c\_2Eset\_relation\_2Ereln\_to\_rel\ A\_27a\ A\_27b)\ V0r)\ V1x)\ V2y)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)) \\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1x)\ V2y))\ V0r)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0R \in ((2^{A\_27b})^{A\_27a}). ((ap\ (c\_2Eset\_relation\_2Ereln\_to\_rel\ A\_27a\ A\_27b) \\ (ap\ (c\_2Eset\_relation\_2Erel\_to\_reln\ A\_27a\ A\_27b)\ V0R)) = V0R)) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). ((ap\ (c\_2Eset\_relation\_2Erel\_to\_reln\ A\_27a\ A\_27b) \\ (ap\ (c\_2Eset\_relation\_2Ereln\_to\_rel\ A\_27a\ A\_27b)\ V0r)) = V0r)) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0r1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). (\forall V1r2 \in \\ (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). (((ap\ (c\_2Eset\_relation\_2Ereln\_to\_rel\ A\_27a\ A\_27b)\ V0r1) = (ap\ (c\_2Eset\_relation\_2Ereln\_to\_rel\ A\_27a\ A\_27b)\ V1r2)) \Leftrightarrow (V0r1 = V1r2)))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0R1 \in ((2^{A\_27b})^{A\_27a}). (\forall V1R2 \in ((2^{A\_27b})^{A\_27a}). \\ (((ap\ (c\_2Eset\_relation\_2Erel\_to\_reln\ A\_27a\ A\_27b)\ V0R1) = (ap\ (c\_2Eset\_relation\_2Erel\_to\_reln\ A\_27a\ A\_27b)\ V1R2)) \Leftrightarrow \\ (V0R1 = V1R2)))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}), \\ ((ap\ (c\_2Eset\_relation\_2Etc\ A\_27a)\ V0r) = (ap\ (c\_2Eset\_relation\_2Erel\_to\_reln \\ A\_27a\ A\_27a)\ (ap\ (c\_2Erelation\_2ETC\ A\_27a)\ (ap\ (c\_2Eset\_relation\_2Ereln\_to\_rel \\ A\_27a\ A\_27a)\ V0r)))))) \end{aligned} \quad (16)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}), \\ ((p\ (ap\ (c\_2Eset\_relation\_2Eacyclic\ A\_27a)\ V0r)) \Leftrightarrow (p\ (ap\ (c\_2Erelation\_2Eirreflexive \\ A\_27a)\ (ap\ (c\_2Erelation\_2ETC\ A\_27a)\ (ap\ (c\_2Eset\_relation\_2Ereln\_to\_rel \\ A\_27a\ A\_27a)\ V0r)))))) \end{aligned}$$