

thm_2Eset_relation_2Eantisym_reln_to_rel_conv (TMK7bLR7NVVdkjZcH8YPqWvyTh9H1f2uTPs)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Erelation_2Eantisymmetric$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21 2) (\lambda V1R \in 2.V1R))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (\lambda V2z \in 2.V2z))$

Definition 8 We define $c_2Ebool_2E_IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Eset_relation_2Eantisym$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (3)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Definition 10 We define $c_2Eset_relation_2Erel_to_rel$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 11 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 12 We define $c_2Eset_relation_2Erel_to_reln$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27b})^{A_27a})$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0xy \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\forall V1R \in ((\\ 2^{A_27b})^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ A_27a\ A_27b))\ V0xy)\ (ap\ (c_2Eset_relation_2Erel_to_reln\ A_27a \\ A_27b)\ V1R))) \Leftrightarrow (p\ (ap\ (ap\ V1R\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0xy)) \\ (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V0xy)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}), (\forall V1x \in \\ & \quad A_27a. (\forall V2y \in A_27b. ((p\ (ap\ (ap\ (ap\ (c_2Eset_relation_2Ereln_to_rel \\ & \quad A_27a\ A_27b)\ V0r)\ V1x)\ V2y)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ & \quad A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))\ V0r)))))) \\ & \hspace{15em} (12) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0R \in ((2^{A_27b})^{A_27a}). ((ap\ (c_2Eset_relation_2Ereln_to_rel \\ & \quad A_27a\ A_27b)\ (ap\ (c_2Eset_relation_2Ereln_to_reln\ A_27a\ A_27b) \\ & \quad V0R)) = V0R)) \\ & \hspace{15em} (13) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}), ((ap\ (c_2Eset_relation_2Ereln_to_reln \\ & \quad A_27a\ A_27b)\ (ap\ (c_2Eset_relation_2Ereln_to_rel\ A_27a\ A_27b) \\ & \quad V0r)) = V0r)) \\ & \hspace{15em} (14) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0r1 \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}), (\forall V1r2 \in \\ & \quad (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}), (((ap\ (c_2Eset_relation_2Ereln_to_rel \\ & \quad A_27a\ A_27b)\ V0r1) = (ap\ (c_2Eset_relation_2Ereln_to_rel\ A_27a \\ & \quad A_27b)\ V1r2)) \Leftrightarrow (V0r1 = V1r2)))) \\ & \hspace{15em} (15) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0R1 \in ((2^{A_27b})^{A_27a}), (\forall V1R2 \in ((2^{A_27b})^{A_27a}). \\ & \quad (((ap\ (c_2Eset_relation_2Ereln_to_reln\ A_27a\ A_27b)\ V0R1) = \\ & \quad (ap\ (c_2Eset_relation_2Ereln_to_reln\ A_27a\ A_27b)\ V1R2)) \Leftrightarrow \\ & \quad (V0R1 = V1R2)))) \\ & \hspace{15em} (16) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}), \\ & \quad ((p\ (ap\ (c_2Eset_relation_2Eantisym\ A_27a)\ V0r)) \Leftrightarrow (p\ (ap\ (c_2Erelation_2Eantisymmetric \\ & \quad A_27a)\ (ap\ (c_2Eset_relation_2Ereln_to_rel\ A_27a\ A_27a)\ V0r)))))) \end{aligned}$$