

# thm\_2Eset\_relation\_2Edomain\_mono (TMHkzdVcd7po9fFSz3GgKds4GSHZJ7qegtn)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 4** We define  $c\_2Ebool\_2EET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 6** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 (2^{A\_27a})))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Ebool\_2E\_2F\_5C (2^{A\_27a})))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 (2^{A\_27a}))))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (3)$$

**Definition 11** We define  $c\_2Eset\_relation\_2Edomain$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). \\ ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (c\_2Eset\_relation\_2Edomain \\ A\_27a\ A\_27b)\ V1r)))) \Leftrightarrow (\exists V2y \in A\_27b. (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a \\ A\_27b)\ V0x)\ V2y))\ V1r)))))) \end{aligned} \quad (4)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). (\forall V1r\_27 \in \\ (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET \\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0r)\ V1r\_27))) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET \\ A\_27a)\ (ap\ (c\_2Eset\_relation\_2Edomain\ A\_27a\ A\_27b)\ V0r))\ (ap \\ (c\_2Eset\_relation\_2Edomain\ A\_27a\ A\_27b)\ V1r\_27)))))) \end{aligned}$$