

# thm\_2Eset\_relation\_2Efinite\_prefixes\_comp (TMJN6vpE8BtBjkNk6z8R1AF3G93CbjaiNXQ)

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**Definition 1** We define  $c\_2Emin\_2E40$  to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then}$  (the  $(\lambda x. x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0 f \in ((A. 27c^{A. 27b})^{A. 27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EC$  to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0 f \in ((A. 27c^{A. 27b})^{A. 27a}))$

**Definition 5** We define  $c\_2Ebool\_2E21$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0 x \in 2. V0 x)) (\lambda V1 x \in 2. V1 x))$

**Definition 6** We define  $c\_2Ebool\_2E21$  to be  $\lambda A. 27a : \iota. (\lambda V0 P \in (2^{A. 27a}). (ap (ap (c\_2Emin\_2E3D (2^{A. 27a})) (\lambda V1 f \in (2^{A. 27a}). (ap V1 f V0 x))))$

**Definition 7** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. \lambda V0 f \in (A. 27b^{A. 27c}). \lambda V1 g \in (A. 27c^{A. 27b}). (ap (ap (c\_2Emin\_2E3D (2^{A. 27a})) (\lambda V2 h \in (2^{A. 27a}). (ap V2 h V0 x))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. nonempty A. 27a \Rightarrow \forall A. 27b. nonempty A. 27b \Rightarrow c\_2Epair\_2ESND A. 27a A. 27b \in (A. 27b)^{(ty\_2Epair\_2Eprod A. 27a A. 27b)} \tag{2}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. nonempty A. 27a \Rightarrow \forall A. 27b. nonempty A. 27b \Rightarrow c\_2Epair\_2EFST A. 27a A. 27b \in (A. 27a)^{(ty\_2Epair\_2Eprod A. 27a A. 27b)} \tag{3}$$

**Definition 8** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A. 27a : \iota. (\lambda V0 x \in A. 27a. (\lambda V1 f \in (2^{A. 27a}). (ap V1 f V0 x)))$

**Definition 9** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (4)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A-27b}}) \end{aligned} \quad (5)$$

**Definition 13** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A-27a}).\lambda V1s \in$

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 15** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c\_2Epred\_set$

**Definition 16** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 17** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 18** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E$

**Definition 19** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A-27$

**Definition 20** We define  $c\_2Eset\_relation\_2Ercomp$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0r1 \in (2^{(ty\_2E$

**Definition 21** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A-27a}).(ap (c\_2$

**Definition 22** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 23** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E\_21 2)$

**Definition 24** We define  $c\_2Eset\_relation\_2Efinite\_prefixes$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge \\ & (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (17)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{.27})) \wedge ((p \ V1x_{.27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{.27})))))) \Rightarrow ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{.27}) \Rightarrow (p \ V3y_{.27})))))) \Rightarrow (18)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (ap \ V0P \ V1a)))))) \Rightarrow (19)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap \ (ap \ (c_{.2Epair\_2E\_2C} \ A_{.27a} \ A_{.27b}) \ V0x) \ V1y) = (ap \ (ap \ (c_{.2Epair\_2E\_2C} \ A_{.27a} \ A_{.27b}) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \Rightarrow (20)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap \ (ap \ (c_{.2Epair\_2E\_2C} \ A_{.27a} \ A_{.27b}) \ V0x) \ V1y) = (ap \ (ap \ (c_{.2Epair\_2E\_2C} \ A_{.27a} \ A_{.27b}) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \Rightarrow (21)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0x \in (ty_{.2Epair\_2Eprod} \ A_{.27a} \ A_{.27b}).((ap \ (ap \ (c_{.2Epair\_2E\_2C} \ A_{.27a} \ A_{.27b}) \ (ap \ (c_{.2Epair\_2EFST} \ A_{.27a} \ A_{.27b}) \ V0x)) \ (ap \ (c_{.2Epair\_2ESND} \ A_{.27a} \ A_{.27b}) \ V0x)) = V0x)) \Rightarrow (22)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow \forall A_{.27c}.nonempty \ A_{.27c} \Rightarrow (\forall V0f \in ((A_{.27c}^{A_{.27b}})^{A_{.27a}}).(\forall V1x \in A_{.27a}.(\forall V2y \in A_{.27b}.((ap \ (ap \ (c_{.2Epair\_2EUNCURRY} \ A_{.27a} \ A_{.27b} \ A_{.27c}) \ V0f) \ (ap \ (ap \ (c_{.2Epair\_2E\_2C} \ A_{.27a} \ A_{.27b}) \ V1x) \ V2y))) = (ap \ (ap \ V0f \ V1x) \ V2y)))) \Rightarrow (23)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p \ (ap \ (ap \ (c_{.2Ebool\_2EIN} \ A_{.27a}) \ V2x) \ V0s)) \Leftrightarrow (p \ (ap \ (ap \ (c_{.2Ebool\_2EIN} \ A_{.27a}) \ V2x) \ V1t)))))) \Rightarrow (24)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in \\ & A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & \quad A\_27a\ A\_27b)\ V0f)))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). \\ & ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\ & \quad A\_27a\ A\_27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0s \in (2^{A\_27a}).((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V0s)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}).(p\ (ap\ (c\_2Epred\_set\_2EFINITE \\ & \quad A\_27b)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a\ A\_27b)\ V1f)\ V0s)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1sos \in \\ & (2^{(2^{A\_27a})}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (c\_2Epred\_set\_2EBIGUNION \\ & \quad A\_27a)\ V1sos)))) \Leftrightarrow (\exists V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a})\ V2s)\ V1sos)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}).(( \\ & p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ (ap\ (c\_2Epred\_set\_2EBIGUNION \\ & \quad A\_27a)\ V0P))) \Leftrightarrow ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ (2^{A\_27a})\ V0P)) \wedge \\ & \quad (\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a})\ V1s)\ V0P)) \Rightarrow (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V1s)))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (35) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (36) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (37) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (38) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (39) \end{aligned}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (41)$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow (\forall V0r1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). \\
& (\forall V1r2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27b\ A\_27c)}). (\forall V2s1 \in \\
& (2^{A\_27b}). (\forall V3s2 \in (2^{A\_27c}). (((p\ (ap\ (ap\ (c\_2Eset\_relation\_2Efinite\_prefixes \\
& A\_27a\ A\_27b)\ V0r1)\ V2s1)) \wedge ((p\ (ap\ (ap\ (c\_2Eset\_relation\_2Efinite\_prefixes \\
& A\_27b\ A\_27c)\ V1r2)\ V3s2)) \wedge (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27b) \\
& (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27b\ A\_27b)\ (\lambda V4x \in A\_27b.(ap\ ( \\
& ap\ (c\_2Epair\_2E\_2C\ A\_27b\ 2)\ V4x)\ (ap\ (c\_2Ebool\_2E\_3F\ A\_27c)\ (\lambda V5y \in \\
& A\_27c.(ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27c) \\
& V5y)\ V3s2))\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c) \\
& (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27b\ A\_27c)\ V4x)\ V5y))\ V1r2)))))))))\ V2s1)))) \Rightarrow \\
& (p\ (ap\ (ap\ (c\_2Eset\_relation\_2Efinite\_prefixes\ A\_27a\ A\_27c) \\
& (ap\ (ap\ (c\_2Eset\_relation\_2Ercomp\ A\_27a\ A\_27c\ A\_27b)\ V0r1)\ V1r2)) \\
& V3s2)))))))))
\end{aligned}$$