

thm_2Eset_relation_2Efinite_prefixes_subset
(TMWqVhKz6DsmTCw7e1E5mnh0q55p3xT33U)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (3)$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 9 We define $c_2Ebool_2E_3I$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\ & ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\ & A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (\\ & ap V0P V1a)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & A_27b. (((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap \\ & (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & A_27b. (((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap \\ & (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ A_27b)\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0x))\ (ap\ (c_2Epair_2ESND \\ & A_27a\ A_27b)\ V0x)) = V0x) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}).(\forall V1x \in \\ & A_27a.(\forall V2y \in A_27b.((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ & A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\ & (ap\ (ap\ V0f\ V1x)\ V2y)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}).(\forall V1v \in \\ & A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b.((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((p\ (ap \\ & (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A_27a}). \\ & (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINTER \\ & A_27a)\ V0s)\ V1t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0y \in (2^{A_27a}).((ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27a)\ (\lambda V1x \in A_27a.(ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ 2) \\ & V1x)\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ V0y)))) = V0y) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & (2^{A_27a}).((ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ (\lambda V2x \in \\ & A_27a.(ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ 2)\ V2x)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\ & (ap\ V0P\ V2x))\ (ap\ V1Q\ V2x)))))) = (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a) \\ & (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ (\lambda V3x \in A_27a.(ap\ (\\ & ap\ (c_2Epair_2E_2C\ A_27a\ 2)\ V3x)\ (ap\ V0P\ V3x))))))\ (ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27a)\ (\lambda V4x \in A_27a.(ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ 2) \\ & V4x)\ (ap\ V1Q\ V4x)))))) \end{aligned} \quad (23)$$

Theorem 1

$$\begin{aligned} \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_{27a}\ A_{27a})}). \\ (\forall V1s \in (2^{A_{27a}}). (\forall V2s_{27} \in (2^{A_{27a}}). (((p\ (ap\ (\\ ap\ (c_2Eset_relation_2Efinite_prefixes\ A_{27a}\ A_{27a})\ V0r)\ V1s)) \wedge \\ (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_{27a})\ V2s_{27})\ V1s))) \Rightarrow ((p\ (\\ ap\ (ap\ (c_2Eset_relation_2Efinite_prefixes\ A_{27a}\ A_{27a})\ V0r) \\ V2s_{27})) \wedge (p\ (ap\ (ap\ (c_2Eset_relation_2Efinite_prefixes\ A_{27a} \\ A_{27a})\ (ap\ (ap\ (c_2Eset_relation_2Errestrict\ A_{27a})\ V0r)\ V2s_{27})) \\ V2s_{27})))))))) \end{aligned}$$