

thm_2Eset_relation_2Efinite_prefixes_subset_rs (TMUDLxAX2gBmv8TuQ3AVq7cZGuexGrMisDo)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define c_2Ebool_2EET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2)$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (\lambda V2z \in 2.V2z))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 10 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 13 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_2Eset_relation_2Efinite_prefixes$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$.

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)}).(\forall V1s \in \\ & (2^{A_27b}).(\forall V2s_27 \in (2^{A_27b}).(((p (ap (ap (c_2Eset_relation_2Efinite_prefixes \\ & A_27a A_27b) V0r) V1s)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET A_27b) \\ & V2s_27) V1s)))) \Rightarrow (p (ap (ap (c_2Eset_relation_2Efinite_prefixes \\ & A_27a A_27b) V0r) V2s_27)))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)}).(\forall V1r_27 \in \\ & (2^{(ty_2Epair_2Eprod A_27a A_27b)}).(\forall V2s \in (2^{A_27b}). \\ & (((p (ap (ap (c_2Eset_relation_2Efinite_prefixes A_27a A_27b) \\ & V0r) V2s)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET (ty_2Epair_2Eprod \\ & A_27a A_27b) V1r_27) V0r)))) \Rightarrow (p (ap (ap (c_2Eset_relation_2Efinite_prefixes \\ & A_27a A_27b) V1r_27) V2s)))))) \end{aligned} \quad (5)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)}).(\forall V1s \in \\ & (2^{A_27b}).(\forall V2r_27 \in (2^{(ty_2Epair_2Eprod A_27a A_27b)}). \\ & (\forall V3s_27 \in (2^{A_27b}).(((p (ap (ap (c_2Eset_relation_2Efinite_prefixes \\ & A_27a A_27b) V0r) V1s)) \Rightarrow ((p (ap (ap (c_2Epred_set_2ESUBSET (ty_2Epair_2Eprod \\ & A_27a A_27b) V2r_27) V0r)) \Rightarrow ((p (ap (ap (c_2Epred_set_2ESUBSET \\ & A_27b) V3s_27) V1s)) \Rightarrow (p (ap (ap (c_2Eset_relation_2Efinite_prefixes \\ & A_27a A_27b) V2r_27) V3s_27)))))))))) \end{aligned}$$