

# thm\_2Eset\_\_relation\_2Ein\_\_rrestrict\_\_alt (TM- PVWWhBzobjkNomLSMmTN5BJjvy5hD16hnQ)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{2}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{3}$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (c\_2Emin\_2E\_3D (2^{A\_27a})) P) P)))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 8** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (4)$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

**Definition 10** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (5)$$

**Definition 11** We define  $c\_2Eset\_relation\_2Errestrict$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a. \\ (\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b) \\ V1q)\ V2r)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2EFST\ A\_27a \\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a \\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.(\forall V2r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V3s \in \\ (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod\ A\_27a \\ A\_27a))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27a)\ V0x)\ V1y))\ (ap\ (ap\ ( \\ c\_2Eset\_relation\_2Errestrict\ A\_27a)\ V2r)\ V3s)))) \Leftrightarrow ((p\ (ap\ (ap \\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ A\_27a\ A\_27a)\ V0x)\ V1y))\ V2r)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ V0x)\ V3s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1y)\ V3s)))))))))) \end{aligned} \quad (11)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod \\ & \quad A_{.27a}\ A_{.27a}). (\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ A_{.27a}\ A_{.27a})}). \\ & \quad (\forall V2s \in (2^{A_{.27a}}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod \\ & \quad A_{.27a}\ A_{.27a}))\ V0x)\ (ap\ (ap\ (c\_2Eset\_relation\_2Errestrict\ A_{.27a}) \\ & \quad V1r)\ V2s))) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod\ A_{.27a} \\ & \quad A_{.27a}))\ V0x)\ V1r)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ (ap\ (c\_2Epair\_2EFST \\ & \quad A_{.27a}\ A_{.27a})\ V0x))\ V2s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ (ap\ (c\_2Epair\_2ESND \\ & \quad A_{.27a}\ A_{.27a})\ V0x))\ V2s)))))) \end{aligned}$$